Cultivating Math in Young Learners

BECOMING NUMERATE

Numeracy across the years of schooling

Students become numerate as they develop the knowledge, skills and dispositions to use mathematics purposefully across their years of schooling K–12, across all learning areas and in their daily lives. Students develop their knowledge, skills and confidence with numeracy as they connect and apply their understanding of mathematics to contexts outside the classroom.

The Early Years Learning Framework (EYLF) emphasises the need for early childhood settings to build on the range of numeracy experiences children have with their families and communities. It emphasises that positive competencies in numeracy are essential for students' successful learning at school.

In the *Australian Curriculum*, numeracy is a general capability, extrapolated in a numeracy learning continuum that builds on from the EYLF. The numeracy learning continuum of the *Australian Curriculum* presents a sequence of learning independent of student age and is labelled from levels 1 to 6, indicating that a particular level typically applies to students by the end of a given year of schooling.

Both *The Early Years Learning Framework* (EYLF) and the *Australian Curriculum* highlight the importance of all teachers acquiring rich mathematical vocabulary and using the language of numeracy in relevant contexts. Numeracy situations are often described in words, so teachers need to support students to analyse the situation to identify the mathematical ideas involved, asking the question: 'can maths help here?' In the EYLF and the *Australian Curriculum* 'becoming numerate' broadly includes understandings about numbers, patterns, measurement, spatial awareness and data.

Numeracy across the curriculum

A commitment to numeracy development is an essential component of learning areas across the curriculum and a responsibility for all teachers.

Students become numerate as they engage with numeracy opportunities and experiences across the learning areas of the *Australian Curriculum* and in learning linked to the outcomes of *The Early Years Learning Framework*. Numeracy happens when students understand the role of mathematics and have the dispositions and capacities to use mathematical knowledge and skills purposefully. Examples of becoming numerate in learning across the curriculum can be found on the *Australian Curriculum* website. The following extract can be accessed from: www.australiancurriculum.edu.au/generalcapabilities/overview/general-capabilities-in-the-learning-areas

In **English** students develop numeracy capability when they interpret, analyse and create texts involving quantitative and spatial information such as percentages and statistics, numbers, measurements and directions.

In **Mathematics** students develop numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is important that the Mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context. A particularly important context for the application of *Number and Algebra* is financial mathematics. In *Measurement and Geometry*, there is an opportunity to apply understanding to design. The twenty-first century world is information driven, and through *Statistics and Probability* students can interpret data and make informed judgments about events involving chance.

In **Science** students develop numeracy capability when they collect both qualitative and quantitative data, which is analysed and represented in graphical forms and through learning data analysis skills, including identifying trends and patterns from numerical data and graphs.

Across **the Arts** subjects, students use spatial reasoning to solve problems involving space, patterns, symmetry, 2D shapes and 3D objects, scale, and proportion and measurement to explore length, area, volume, capacity, time, mass and angles.

In **Technologies** students cost and sequence when making products and managing projects. They use three-dimensional models, create accurate technical drawings, work with digital models and use computational thinking in decision-making processes when designing and creating best-fit solutions.

In **Health and Physical Education** students use calculation, estimation and measurement to collect and make sense of information related to nutrition, fitness, navigation in the outdoors or various skill performances. They use spatial reasoning in movement activities and in developing concepts and strategies for individual and team sports or recreational pursuits.

In **Languages** there are opportunities for learners to use the target language to develop skills in numeracy including processes such as using and understanding patterns, order and relationships to reinforce concepts such as number, time or space in their own and in others' cultural and linguistic systems.

In **Work Studies** (Years 9 and 10) students strengthen their numeracy skills by making direct connections between their mathematical learning and the nature of mathematics required in workplaces and enterprises. Students recognise that financial literacy is a requirement across enterprises and that numeracy helps them manage salaries and personal and workplace budgets and calculate personal and enterprise tax liabilities.

Humanities and Social Sciences

- In **History** students develop numeracy capability as they learn to use scaled timelines, including those involving negative and positive numbers, as well as calendars and dates to recall information on topics of historical significance and to illustrate the passing of time.

- In **Geography**, students develop numeracy capability as they investigate concepts of location and distance, spatial distributions and the organisation and management of space within places; in constructing and interpreting maps, students work with numerical concepts of grids, scale, distance, area and projections.

- In **Economics and Business** (from Year 5), students use numeracy to understand the principles of financial management, and to make informed financial and business decisions.

- In **Civics and Citizenship** (from Year 3) numeracy knowledge and skills are applied to analyse, interpret and present information in numerical and graphical form, including conducting surveys and representing findings in graphs and charts.

WHOLE SCHOOL APPROACHES TO NUMERACY

Numeracy is regarded as one of the key dimensions of all learning. Increases in numeracy outcomes will only occur when there is a sustained whole school commitment to systematic curriculum delivery over a period of time. Improvement in numeracy achievement requires a whole school commitment to the following key aspects:

Organisation

A culture of collaboration empowers staff to work together on numeracy provision, discussing, reflecting, planning, setting goals, developing resources, analysing data and work samples, and sharing learning. An effective numeracy improvement strategy benefits from leadership by a numeracy leader or a numeracy team in managing and leading:

- planning and review cycles

- collecting and analysing data

- setting targets

- ensuring coherence and continuity across the years

- promoting formative assessment strategies to guide numeracy teaching

- building staff capacity

- enabling the sharing of effective numeracy practices across the school

- supporting the principal on the development of whole of school interventions for students requiring differentiated numeracy support

- identifying resource needs and allocation

- convening and structuring year group numeracy planning meetings.

Planning

It is important for schools to allocate time for early years and grade meetings so that teachers can plan how to integrate numeracy activities into learning experiences and plan units of work for the explicit teaching of the mathematical skills needed for numeracy. Teachers collaboratively decide on consistent approaches to numeracy practices and assessment, share and confirm numeracy language, and develop numeracy resources to support all learners.

Principals and the leadership teams have a role in supporting teachers to understand numeracy development as described in the EYLF, and the progressive development of numeracy across the years of schooling as identified in the *Australian Curriculum* numeracy continuum and *Australian Curriculum: Scope and Sequence for Mathematics.*

Teacher planning includes differentiating the curriculum for students requiring additional numeracy support including students who are gifted and talented and those who are not making expected progress. Refer to *Good Teaching: Differentiated Classroom Practice* and *Supporting Literacy and Numeracy Success.*

Teaching and learning

An inquiry approach to teaching and learning numeracy is recommended for all years of schooling.

Thinking that is productive, purposeful and intentional is at the centre of effective learning (ACARA, Critical and Creative Thinking general capability). All students need be explicitly taught skills to solve problems across the curriculum. As they become numerate, students use their mathematical understandings, skills, dispositions and strategies to solve problems they meet in a wide range of contexts.

At all levels of schooling students need to recognise which aspects of their mathematical knowledge are relevant to a particular situation, select and then apply those ideas and skills. This involves problem solving skills and higher order thinking skills, for instance, when cooking, planning a trip or managing finances. The fundamental numeracy question is: 'what maths will help here?'

Because numeracy involves context, it is usually described in words. For young students to understand about how context changes the numeracy demands of a situation we need to teach them to visualise and paraphrase, to see 'in their mind's eye' what the words are saying and learn to talk about it in their own words. For example – 'If we need to share the birthday cake among our class, how many pieces will we need? How will we cut the cake fairly? Let's cut it in half and then in half again.' Visualising and paraphrasing are the first steps to understanding and clarifying a situation and then solving a problem. Language, discussion and planned mathematical conversations support students in developing their numeracy.

The mathematics that underpins numerate behaviours is first learned as a body of knowledge in the primary years of school. By understanding a context and determining that 'some mathematics will help here', students learn to make choices about *what* mathematics will help, and *what* strategies they will use to apply the mathematics chosen.

Students can then apply the mathematics and strategies chosen confidently and make a judgement about whether their solution makes sense in the context. If their solution *does* make sense, they gain confidence in their application of mathematics and are more likely to choose to use mathematics next time they identify it as being needed. Continued success will mean students become increasingly numerate in many different contexts.

To be confident problem solvers, students need to develop favourable attitudes towards applying their mathematics knowledge and be able to make strategic choices about which concepts and methods to apply in which contexts. This requires careful planning for tasks which promote problem solving, thinking and inquiry (see *Good Teaching: Differentiated Classroom Practice* and *Supporting Literacy and Numeracy Success*).

Problem solving is one of the mathematical proficiencies that should be used in teaching mathematics. However, the key numeracy question *will some Maths help here*? is usually asked outside of, or prior to, mathematics lessons. Hence it is part of general problem solving which can be used in every context (including other learning areas) and not just in the teaching and learning of mathematics.

The emphasis in the early years of schooling is on teaching the mathematics that students can draw on when confronted with a situation where some mathematics can help. Teachers in these years focus on supporting students to learn their mathematics in familiar contexts and to interpret the contexts by visualising and paraphrasing, as described earlier. These skills are part of a problem solving strategy which all students should be taught as part of their numeracy development.

Problem solving begins from the early years and teachers and educators use the EYLF to plan play-based mathematical learning experiences which enable children to: be confident and involved learners (Outcome 4); develop their emerging autonomy, interdependence resilience and sense of agency (Outcome 1); as well as paying attention to the mathematics-specific content described in Outcomes 3, 4 and 5. Teachers gradually transition from the EYLF to the content of the *Australian Curriculum* as students demonstrate the requisite skills and understandings. Problem solving experiences also support students as they develop the general capability of critical and creative thinking.

Effective numeracy teachers:

1. Explicitly teach mathematical language and the concepts that the words describe.

2. Scaffold from non-mathematics (common) understandings to precise mathematical ones.

3. Explicitly teach mathematical concepts needed for numeracy in ways that modify common or informal understandings and developing deep mathematical understandings.

4. Teach students mathematics as concepts, not methods. This involves students using words – initially through talk alone and later included in their writing – to understand what they are doing. Teaching mathematics *skills* alone is insufficient to develop the general capability of numeracy.

In summary, to problem solve effectively in numeracy, students need to know how to:

1. **Clarify** the problem.

2. **Choose** the mathematics, tools, procedures and/or skills required.

3. **Use** and or apply what has been chosen.

4. **Interpret** and check if the solution worked.

5. **Communicate and talk** about the steps they took to reach the solution (i).

Students need to be explicitly taught these steps if they are to be numerate. Knowing mathematics is essential, but it is not sufficient.

Support for teachers

- Teachers have varying levels of experience and expertise in different aspects of numeracy education and they therefore require different levels of professional support.

- Having a numeracy leader or a numeracy team who, along with senior staff, can work with teachers 'shoulder to shoulder', as well as identifying their professional learning needs underpins school improvement in this area of the curriculum.

- Professional learning is more effective when it is student focused, data informed and sustained rather than in one-off sessions.

- Teachers need opportunities to learn about the mathematical ideas that underpin numeracy and the evidence-based teaching approaches that support student learning.

- Teachers need to be able to articulate both what they do and why they do it.

When these professional supports are in place and when effective numeracy practices are shared across the school, whole school improvement is sustained.

Assessment

To ensure continuity of numeracy development, it is important to develop a consistent approach to assessment, as outlined in *Good Teaching: Quality Assessment Practices*.

- Whole school practices for collation of data and reporting procedures support planning and tracking of student achievement.

- Teachers and leaders work together to investigate patterns of students' strengths or underachievement and plan for interventions based on information from the data. Refer to *Supporting Literacy and Numeracy Success*.

- Assessment should lead to more effective teaching with teams developing a plan of action and selecting focus areas for improvement.

- Progress is monitored and teaching is adjusted accordingly.

- Early years and year group teachers benefit from sharing formative numeracy assessment practices and planning for adjustments to teaching as a result of new understandings of learners.

- Success criteria should be shared with students who increasingly take responsibility for addressing the criteria and assessing their own numeracy progress.

NUMERACY K–2

This K–2 resource highlights the importance of numeracy in the early years and identifies the main mathematical concepts and understandings students need in order to become numerate as they engage in learning across the curriculum. It also underlines the importance of teachers acquiring rich mathematical vocabulary to explain students' ideas and support their numeracy development.

The numeracy environment

As *The Early Years Learning Framework* explains, numeracy is best nurtured in a classroom climate that allows children to develop dispositions for learning such as curiosity, cooperation, confidence, creativity, commitment, enthusiasm, effort, persistence and imagination, along with skills and processes such as problem solving, risk-taking, inquiry, experimentation, hypothesising, researching and investigating (EYLF).

A classroom setting that encourages numeracy learning includes:

- learning spaces designed to facilitate whole class, group, pair and individual work

- numeracy materials organised for independent learning (e.g. counters, dice, bottle tops, pebbles, gumnuts, buttons, pegs, measuring tools, calculators)

- opportunities for experiential hands-on learning

- various technologies (e.g. individual mini-whiteboards, tablets, laptops, visualiser to project student work)

- teacher and student made materials and posters (rather than commercial materials) that have a meaningful connection to the curriculum and are effective tools for teaching and learning

- frequently referenced materials e.g. maths dictionaries, number charts, number lines, numeral cards

- displays that are fresh, uncluttered and purposeful

- student work on display that shows their thinking as well as their conclusions.

Planning and teaching for numeracy

Kindergarten teachers engage with *The Early Years Learning Framework (ELYF)* to understand what students are expected to know, do and understand. In Foundation (Prep) students are transitioning between the EYLF and the *Australian Curriculum* and teachers refer to both sources according to their students' level of development and capability. In Years 1 and 2 teachers connect numeracy learning to the Numeracy general capability of the *Australian Curriculum*, underpinned by Foundation (Prep) and Years 1 and 2 of the *Australian Curriculum: Mathematics.*

Planning also requires teachers to have understanding of what their students already know, understand and can do, so that they can build on from this learning into the intended learning in K–2. Teachers need to gather and scrutinise diagnostic data (e.g. from the AEDC and system data such as that collected through PIPS assessments and the Kindergarten Development Check) and to talk to parents and carers, as well as to other teachers or support staff in the school who know the students and their families.

Effective planning and teaching emphasises backward design and the importance of clear links between learning goals and assessment tasks (refer to *Good Teaching: Curriculum Mapping and Planning*).

It includes:

- Accessing data about each student through analysis of previous assessments such as PIPS, Kindergarten Development Check, other formal assessments and previous reports.

- Assessing students' current understandings about concepts through pre-assessments (mainly through oral questioning and inviting input from parents about home experiences).

- Being clear about the mathematics that needs to be learnt by backward mapping from curriculum outcomes (both the achievement standards and content descriptors from the *Australian Curriculum: Mathematics* and/or the expected indicators the numeracy continuum or outcomes that form the EYLF).

- Determining the scaffolding, explicit teaching and tasks needed to bridge prior understandings and new numeracy and mathematical ideas.

- Determining contexts to teach the numeracy in familiar and engaging ways that promote discussion and focus on problem solving.

- Facilitating intentional, play-based learning with planed opportunities for explicit focus on the mathematical ideas needed for numeracy.

- Teaching mathematics and numeracy in ways that promote understanding, support fluency and demand reasoning and problem solving.

- Determining the tasks required for students to demonstrate their understanding of numeracy.

- Generating data for diagnosing future learning and intervention needs, such as early screening, diagnostic assessment, benchmarking, outcomes assessment, success criteria (including rubrics), observation checklists, portfolios of student work, parent interviews (see Appendix).

- Being clear about learning goals and planning assessment tasks linked to the goals before planning the learning experiences which will helps students achieve these goals.

- Determining the materials and models which will be support learners in building mental models of the mathematical ideas e.g. manipulatives, number lines, number charts etc.

- Considering ways to differentiate the learning to meet individual student's needs.

Monitoring and assessment

In order to monitor the learning against what the students are expected to learn teachers need to refer back to the intended learning in the *Australian Curriculum* or the EYLF. Teachers need to reflect on what they want students to learn to remind themselves of their precise learning goals for individuals and groups of students. Assessment must focus on assessing the intended learning as opposed to what teachers believe they have taught—how far have students travelled in relation to my expectations for their learning? Further support and elaborations on formative and summative assessment and the importance of feedback to students is found in the DoE publication *Good Teaching: Quality Assessment Practices.*

It is not the purpose of this book to describe and explain in detail to teachers how to assess the mathematics learned by their students if they follow the approaches described here. Assessing whether students deeply understand the numbers, operations and strategies they have been taught and whether students can use them to solve problems and estimate, requires teachers to assess *whether students can make choices about which of these to use*. If a teacher has spent a week teaching addition of double-digit numbers and then assesses this skill, students are merely demonstrating that they have learned what has been taught, rather than what they have learned in a deep, lasting and transferable way.

To assess the deep understandings of mathematics, as described above, teachers must ensure that they have taught the understandings to all students using a high expectations approach and differentiation strategies as described in *Good Teaching: Differentiated Classroom Practice.*

Monitoring mathematics learning should be done by determining, at the point in time of the particular assessment, the extent to which students know and understand a specific content descriptor and the associated aspects of the achievement standard or the outcomes of the EYLF; this indicates what support they then need in order to move to the next level. For example, if students can only *recall* then they need targeted teaching to help them understand the concepts of addition and subtraction. If they are unable to understand a context sufficiently well to choose the appropriate operation from the words given and explain this choice, then they need targeted support to visualise the problem and think through alternative strategies and solutions.

If students don't appear to be progressing it is important to reflect on your assessment tasks:

• Are the questions appropriate to the task?

• Do they assess the learning that you planned for?

• Are there words and phrases in the tasks that students may not be able to read and understand?

• Do your observations validate the learning or are you seeing different things in students' behaviours than you are seeing in written tasks?

It is helpful to ask a peer or mentor check the alignment between the tasks, the intended learning, and the teaching you have delivered. In a cyclical fashion, teachers then review how well students dealt with a task and modify subsequent learning challenges to ensure all students can engage, succeed and progress.

Past Year 3 NAPLAN Numeracy test papers offer a good source of assessment questions for students in Year 2. While it provides only one way to assess numeracy alongside everyday teacher-student interactions, the NAPLAN questions generally align closely with the *Australian Curriculum* definition of numeracy and:

• provide no hints about the mathematics students should choose to use

• are for the most part written in contexts so that students have to understand these before choosing the mathematics to use.

NUMERACY KEY ELEMENTS K–2

The key organising elements for numeracy used in the *Australian Curriculum* numeracy general capability are:

- Estimating and calculating with whole numbers

- Recognising and using patterns and relationships

- Using fractions, decimals, percentages, ratios and rates

- Using spatial reasoning

- Interpreting statistical information

- Using measurement.

It is important to note that the verbs used in the first numeracy element indicate higher order cognitive functions. Words such as es*timate* and *calculate* generally require higher level thinking than others used in the *Australian Curriculum: Mathematics* content descriptors, such as recall, investigate, count and develop. The words in the numeracy element reinforce for teachers the depth of learning of mathematics students will need in order (to choose) to apply mathematics to contexts outside the mathematics classroom.

The following sections A–F elaborate the significant aspects of numeracy and mathematics that teachers should focus on with young students in the early years. There are key messages for teachers, explanations of important mathematical terms and practical suggestions about how to put the ideas into practice.

A. Estimating and calculating with whole numbers

The numeracy general capability identifies 'estimating and calculating with whole numbers' as a key element. 'This element involves students using numbers for different purposes. Students apply skills in estimating and calculating with whole numbers to solve and model everyday problems in a wide range of authentic contexts using efficient mental, written and digital strategies'. An estimate is a close approximation of the correct answer and it involves thought and calculation.

Key messages

The key message for teachers is that for numerate behaviour, all calculation requires an element of estimation at the outset. In order to determine that a solution makes sense in context we need to first have a sense of the nature of the solution we are expecting.

While our ultimate goal may be to develop students' rigour and fluency in mental computation, in the early years, the emphasis should be on establishing deep understandings about numbers and how they work. Students rarely deepen their understanding about arithmetical operations by practising algorithms (ii). Therefore, effective teachers should focus on developing deep understanding of number in problem solving contexts, rather than teaching algorithms in isolation.

In teaching students in K–2 how to estimate and calculate with whole numbers effective teachers ensure that students deeply:

1. Understand what whole numbers are and that they can be represented in words, numerals, objects, drawings and on number lines.

2. Understand operations and how they can be used to represent and solve problems.

3. Understand how to break down and partition whole numbers to help solve problems.

As the *Australian Curriculum: Mathematics* strand Number and Algebra makes clear, students during the years F–2, increase the size of numbers they can work proficiently with, so that

in Prep students can initially do these things with whole numbers from zero to ten, and then to twenty. In Years 1 and 2 students work from 100 to 1000, building their understanding of how the number system works.

Links to the curriculum

Links to *The Early Years Learning Framework*:

These understandings about number link particularly to the EYLF Outcome 5: 'Children are effective communicators' and the key component: 'Children begin to understand how symbols and pattern systems work'. Children will also engage with the other EYLF outcomes as they investigate and explore the ideas of number through play and planned inquiry.

Links to the *Australian Curriculum: Mathematics*

These understandings about number link with the Number and Algebra strand, Foundation (Prep) to Year 2 and with the Numeracy learning continuum Levels 1 and 2, *Estimating and Calculating with Whole Numbers*, as shown below:

EYLF	Australian Curriculum: Mathematics		
Kindergarten	**Foundation (Prep)**	**Year 1**	**Year 2**
Outcome 5 *Children are effective communicators:* Children begin to understand how symbols and pattern systems work **Outcome 4** *Children are confident and involved learners:* Children develop a range of skills and processes such as problem solving, inquiry, experimentation, hypothesising, researching and investigating	**Relevant content descriptors from the Number and Algebra Strand**		
	• Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point • Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond • Subitise small collections of objects • Represent practical situations to model addition and sharing • Compare, order and make correspondences between collections, initially to 20, and explain reasoning	• Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero • Recognise, model, read, write and order numbers to at least 100. Locate these numbers on a number line • Count collections to 100 by partitioning numbers using place value • Represent and solve single addition and subtractions problems using a range of strategies including counting on, partitioning and rearranging parts • Recognise, describe and order Australian coins according to their value	• Investigate number sequences, initially those increasing and decreasing by twos, threes, fives and tens from any starting point, then moving to other sequences • Recognise, model, represent and order numbers to at least 1000 • Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting • Explore the connection between addition and subtraction • Solve simple addition and subtraction problems using a range of efficient mental and written strategies • Recognise and represent multiplication and repeated addition, groups and arrays • Recognise and represent division as grouping into equal sets and solve simple problems using these representations • Count and order small collections of Australian coins and notes according to their value

EYLF	Australian Curriculum: Mathematics		
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
	• Students make connections between number names, numerals and quantities up to 10 • They count to and from 20 and order small collections	• Students describe number sequences resulting from skip counting by 2s, 5s, and 10s. They count to and from 100 and locate numbers on a number line • They carry out simple additions and subtractions using counting strategies. They partition numbers using place value • They recognise Australian coins according to their value	• Students recognise increasing and decreasing number sequences involving 2s, 3s, and 5s • They represent multiplication and division by grouping into sets. They identify the missing element in a number sequence • They count to and from 1000 • They perform simple addition and subtraction calculations using a range of strategies. They associate collections of Australian coins with their value
	Australian Curriculum **Numeracy Continuum**		
	Level 1a	**Level 1b** Typically by the end of Foundation (Prep)	**Level 2** Typically by the end of Year 2
	Estimating and calculating with whole numbers		
	• Demonstrate concepts of counting using every day experiences • Recognise the effects of adding to and taking away from a collection of objects • Identify situations that involve the use of money	• Connect and order number names, numerals and groups of objects using numbers up to two digits • Solve everyday addition and share stories • Recognise the different value of coins and notes in the Australian monetary system	• Model, represent, order and use numbers up to four digits • Estimate the solution to a problem and then calculate the answer • Identify and use combinations of coins and notes for simple purchases

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

1. Teaching students to deeply understand what whole numbers are and that they can be represented in words, numerals, objects, drawings and on number lines

Teaching what whole numbers are begins with counting. Counting is needed for quantifying since by counting we can determine 'how many'. However, many students can tell 'how many' just by looking at a quantity of similar objects or at a collection of different objects; they can *subitise*.

Subitising is *seeing* 'how many' by merely *looking* rather than counting. It develops before counting and one-to-one correspondence. Because subitising is a visual activity it supports the understanding of whole numbers and quantifying — for example, that three is one more than two and six is two more than four. Subitising helps students to think of numbers in parts, they can see that five is four and one more, that eight is six and two more, and nine is three and three and three.

By supporting students to see numbers made up of component parts, subitising can assist them to partition numbers, a highly useful skill when operating with numbers.

Some activities that help students to understand that numbers represent quantities include:

- Showing students flash cards with small objects randomly placed or many objects arranged in groups, and asking: *how many are there*?

- Having students work in pairs with a small handful of pebbles, shells or knucklebones; one student drops some on the mat and the other quickly says how many there are.

To teach the different representations of numbers teachers begin with the words common to all young students 'one, two, three….' and so on, and provide lots of experiences to ensure students have one-to-one correspondence in their counting and aren't just saying the words in order. Ask students to move one object at a time as they say the counting words in order. They should practise starting from any numeral – not always 'one'. Having students doing this independently one-on-one with you is the ideal way to assess their deep learning of this understanding. Students should also learn that when they say 'one' as representing one object of the first number said when counting, it can be represented as a written or spoken word or as a symbol '1', (which we call a numeral). Later, they can be asked to show this in a table such as

Word	Numeral	Objects/ Drawing
six	6	

For one-on-one assessment, students might for example, be given a box of objects and asked to show you five of them. Ensure that objects/drawings are not just of school shapes such as counters but that they include things that are familiar to them from home and school. The word 'zero' or the symbol '0' tells us there are no objects or drawings or quantities; we don't refer to zero as 'nothing'.

As students become competent with these representations and can move from one to another, teachers introduce the concept of a number line. Facility with number lines will be very useful for students when they begin operating with numbers as an unmarked number line can replace the need for many written algorithms, particularly when adding and subtracting (iii).

Using experiences with early 'addition' students can 'add one' each time to obtain more. They can 'build' a number line by all standing in a row holding a card which has their 'next' number on it. Encourage them to say: *I'm number one, I'm number two*, and so on – different each day. They should also be encouraged to go backwards to zero. Have some students 'sit out' this first activity and then ask the class: '*Where should this number go?*'

For older students, the concept of 'equidistance between numbers' is important. Stress this by having students draw their own number lines that go forwards or backwards from a given number. Students should also be able to 'guess' a number on an unmarked number line by using the distance between given numbers. For example, on the number line below, two numbers, 12 and 20 are marked, but that is all. Have students work in groups or pairs to guess the missing number and to justify or explain why they think that is the number.

Place value

Counting large collections of objects can be facilitated by partitioning numbers using place value. The way we write numbers is based on groupings of ten. It is important the students understand the 'countableness' of numbers as distinct units between 1 and 10 before they are introduced to place value. That is, when counting seven for example, they know they have seven distinct units rather than one '7'.

Initially, the best way to teach understanding of place value is to focus on the first 'family' in the number system; the relationship between ones, tens and hundreds. This family is called the *ones family*. Using plastic straws is a simple and effective strategy. Students use one straw to represent 'one'. They then count ten straws and put an elastic band around them. By doing this they can see that there are ten single straws in their one bundle of ten – the only thing that is changed is that they are now together in a group of ten, ten of these is one of these.

The benefit of using straws rather than pop sticks is that when you introduce tenths you can simply cut a straw into ten equivalent sections and students can see that the position to the right of the decimal point is ten times smaller.

'Ones family'		
H	T	O

Some activities for promoting depth of understanding about place value learning include

- Have students count the number of stones in a large jar, using a H T O chart. If they use a pencil they can write 'I' in the tens column each time they have ten 'I's and then erase the 'I's:

H	T	O
	III	IIIIIIII
	3	8

Have students write the total number using numerals in the final row of the chart. Give them a range of numbers to make, some requiring grouping and some not.

- Have students work in pairs to count the number of matchsticks (non-ignitable!) in a box. They arrange their matchsticks in groups of ten and record them using a H T O chart. They compare their final number with other students and check that the total is right by counting groups of tens and single ones.

- Show groups of tens and single ones quickly to students on flash cards or digitally. Have students see at a glance how many there are by counting groups and singles and writing their answer.

- Write a two-digit number on the board and ask students to make the number using their straws in groups and singles. Have them repeat this strategy in pairs; one writing the number, the other making the number shown.

- Write a two digit number on the board and show students some straws in groups and singles. Ask questions such as: *How many more straws do I need to make the number on the board? Can I group them to make them easier to count? How?*

- Have pairs of students make a two-digit number using straws e.g. 36 = 3 groups of ten straws and six single straws. Encourage them to think of these straws using non-standard groupings (partitions) such as two bundles of ten straws and 16 single straws. They should put these in a table:

H	T	O
	3	6
	2	16
	1	26

Have them check that there are still 36 in each row by grouping their straws again.

- Repeat the above activities following teaching students that ten groups of ten straws make one hundred.

Students need to be explicitly taught that in each multi-digit number (e.g. 35, 237, and 108) each numeral has a face value and a place value. The face value tells them the value of the *face* or 'how many', and the place value tells them the value of the place (i.e. tens, hundreds or units). Teachers should stress this difference since many students only learn the face value; in the number 43 for example, they will think that the '4' represents four ones instead of four tens. Teachers should stress that in a number with a zero in it, the face value is zero but this is not nothing; the place of the zero tells us that 'there are no ones or no tens in that particular number'.

> When many teachers went to school they were taught *hundreds, tens* and *units*; for many years we have been using *hundreds, tens* and *ones* since units have other uses in mathematics that can confuse children e.g. countable units, a whole *unit* in fraction.

2. Teaching students to deeply understand operations and how they can be used to represent and solve problems

Some contexts for addition and subtraction involve activities that occur in students' homes (e.g. counting the number of sandwiches for lunch) and arise in stories that are read to students. Students can hear and use the words 'altogether', 'total', 'take away', 'more than', 'difference' and other common language that, in context, can mean *add and subtract*. It is important that students learn these *concepts* in play situations before being introduced to the mathematical – and often abstract – terminology. Stories with numbers in them such as *The Three Bears*, *Three Little Pigs*, *Snow White and the Seven Dwarfs* provide engaging contexts for questions such as: *How many chairs were left when the Papa Bear broke one? How many dwarves were there altogether when Dopey and Sleepy met up with the other five?* Students should also be encouraged to write or draw and ask their own number questions for other students in the class to answer.

Just as it is important that students learn to 'trust the count' (i.e. know that the total doesn't change no matter which way the objects are counted) so too they need to understand that adding numbers together will always result in the same number, regardless of the order in which they are added. Teachers should also focus on using *common language* used in addition and subtraction contexts. From contexts outside school, students are familiar with numeracy questions and phrases such as: *How many more than? What is the difference? How many will be left over? What is the total? How many altogether? How many remain? Three more than; Five less than; Who has the most? Who has the least?* If students are encouraged to use everyday language rather than only use mathematical words like 'add' and 'subtract', they will connect their mathematics learning to life outside school.

The symbols representing addition and subtraction should be introduced with reference to common words such as *total*, *altogether*, *difference* and *more*, since these are words that explain and describe common usage of the mathematical concepts. Students need to be explicitly taught the connections and learn how to translate between one representation and another as indicated in the table:

Note that children need to be explicitly taught what a **'number sentence'** is. Teachers should use the words both in oral and written forms consistently so that children learn to read and write them and know what they mean.

Concept	Phrases that are used	Symbol
Addition	How many altogether?	+
	What is the total?	+
	Combine the...	+
	What is the sum?	+
Subtraction	How many more than?	-
	How many are left?	-
	How many are left over (iv)?	-
	How many will remain?	-
	What is the difference between?	-
	How many went away?	-

This knowledge is best taught and assessed in contexts which require students to reason about the words and what they mean. For example: *If Shelley has three more dolls than Jill who has two dolls, how many does Shelley have? And if Sam has twice as many toy cars as Jim, how many does he have?*

Given a number of either similar or dissimilar objects, students in a group can be asked to make up (and write if they are able) ten problems about combining and/or separating their objects. Their problems should be written in words and as number sentences using symbols. They can solve their problems and use

an '=' symbol to indicate that they have added or subtracted to arrive at their solution. Students should be able to write a number story given a number sentence and vice versa. For example:

Number story
If Jill who has two dolls and Shelley has three more than Jill, how many does Shelley have?
Number sentence
2 + 3 = 5

> Note that unlike for addition and subtraction, students should be taught that the symbol representing the words 'is equal to' (i.e. '=') is *not an operator symbol*. In other words, when they see the symbol '=' it does not mean 'do something' but merely describes the relationship between the numbers on either side. This will become clearer following the discussion below about partitioning

3. Teaching students how to break down and partition whole numbers to help solve problems

Partitioning is about breaking numbers down into parts and also building them back up (v). Knowing basic number combinations is essential for mathematical fluency. Early understanding that a *whole* is made up of *parts* starts when students learn to count by first subitising (see p. 16).

As well as supporting the development of counting, subitising is useful in developing *additive thinking*. This is the thinking underpinning the breaking down of numbers into parts which is helpful for students when solving problems using addition and subtraction strategies.

Students should be given activities to help them understand that any number can be thought of as a sum of or difference from other numbers. By visualising ten as four and six for example, students build understanding that the sum of the parts is always equal to the whole, and that when we subtract one of the parts, it is always equal to the other part. These understandings

are necessary for addition, subtraction and grasping the inverse relationship between the two operations (i.e. that addition undoes subtraction and vice versa).

By thinking of 10 as 7 and 3, or as 4 and 6, students can more easily learn *number facts* such as:

3 + 7 = 10 and 10 − 7 = 3 and 10 − 3 = 7

4 + 6 = 10 and 10 − 6 = 4 and 10 − 4 = 6

3	11
14	

This type of thinking is sometimes called *part-part-whole* thinking. It can be used with any pair of numbers. For example, thinking of fourteen as made up of different pairs of numbers can greatly facilitate additions and subtractions of this whole number (14) and its parts.

This thinking is particularly useful for students as the numbers they are working with become larger.

37   63

100
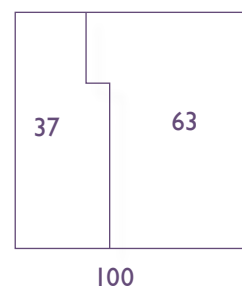
For example, by thinking of one hundred as 63 and 37, children can see that the sum of the parts (37 + 63) is the same as the whole, 100. They can also understand that 100 − 37 = 63 and that 100 − 63 = 37. So, if they start with 63 and add 37 and get 100, then to get *back* to 63 they subtract 37 away from their answer.

Some activities that build this fluency include:

- Ask students to hold their hands up to show different combinations totalling ten fingers. They can work together to find how many different combinations there are.

- Give students four beads and ask them to gather more of the same colour beads to make a total of 7, then 11, then fifteen.

Additive thinking is very useful for mental computation, especially if students deeply understand place value and that adding or subtracting ten from a two-digit number results in changing the digit in the tens column but the digit in the ones column staying the same. For example, for adding 25 to any number students merely think of it as adding ten, adding another ten, and then adding five. Similarly, for subtracting 43 from any number they simply think of it as subtracting twenty, subtracting another twenty, and then subtracting three. They can do this in their head or using an unmarked number line (referred to earlier). For example:

	Think….	On an unmarked number line
Add 14 and 12	Start at 14, add ten, add two	14 ⟶ 24 26
Add 38 to 23	Start at 23, add 40, take two	23 33 43 53 61 63
Subtract 17 from 56	Start at 56, take ten, take 7	39 46 56
Subtract 54 from 73	Start at 73, take fifty, take 4	19 23 73

Whilst a number line can help students perform addition and subtraction, a strong additive thinker may not need to use one to support their thinking; their understanding of place value is sufficient. For example, in adding 38 to 23 they merely *think* 'thirty add twenty is fifty, eight add three is eleven which is ten add one, so 38 add 23 is fifty add ten add one'. Most additive thinkers work from left to right rather than right to left, which most standard algorithms promote.

Teachers should aim for all students to be able to do this sort of thinking – at least for single and double-digit numbers – without any need

for writing algorithms. If students need the support of a pen and paper to jot down parts of their thinking as they go, that's fine, but when they have the deep understanding needed to estimate and solve problems, students will be able to complete these tasks mentally.

From additive to multiplicative thinking

When teaching place value many students will see and understand that counting 'lots of' or 'groups of' ten, for example, is the same as repeatedly adding ten. This repeated addition of the same amount is an important foundation for multiplication. However, if students *continue* to

only think of multiplication as repeated addition, using addition and counting strategies to multiply, they will not deeply understand multiplication. This often leads students relying on algorithmic procedures for multiplication rather than using mental multiplication strategies.

Making the jump from addition to multiplication can be very challenging for some students since they can't necessarily 'see' or 'touch' what is happening. Teachers shouldn't just 'rush in' using terms such as *times* or *multiplied by*; using '*groups of*' is a necessary scaffold for teaching multiplication (vi). One of the best ways to promote the move on from additive to multiplicative thinking is by using a number line. Students can skip count along a line in 3s for example, and then cut the line into equal lengths of three, placing them one on top of the other in an array to show 'groups of 3'.

Students need to learn that numbers can be broken down and rearranged into equal groups and that doing this does not change the total quantity. Teachers should spend many hours having students break down quantities into equal groups (factors). If they have twelve counters each for example, they can work in pairs to find how many groups of the same number there are in twelve:

12 is 3 groups of 4   and   12 is 6 groups of 2

12 is 2 groups of 6   and   12 is 4 groups of 3

Students should be taught that just as we have a symbol for addition '+' and for subtraction '-', we also have a symbol for groups of: 'x'. So, they can write each of the above again and replace the words 'groups of' with 'x':

12 is 3 groups of 4   and   12 is 6 groups of 2
12 = 3 x 4   **12 = 6 x 2**

You can teach students the *commutativity* of grouping (multiplication) by drawing attention to particular groups and counting to show that there are 12 in each grouping, since they can see that:

$12 = 6 \times 2$ and $12 = 2 \times 6$; 'Twelve is six groups of two, and twelve is two groups of six'.

$12 = 3 \times 4$ and $12 = 4 \times 3$; 'Twelve is three groups of four, and twelve is also four groups of three'.

Students should be given many tasks and activities such as the one above, enabling them to deepen these understandings. For example, following a shared activity where students work with concrete materials in pairs or in a group, they can be asked to fill in tables like the one below:

Picture of groups	In words	Using symbols
	20 is five groups of four	$20 = 5 \times 4$
	20 is four groups of five	$20 = 4 \times 5$
	20 is ten groups of two	$20 = 10 \times 2$
	20 is two groups of ten	$20 = 2 \times 10$

Then, arranging groups in rows as an **array** to make counting and 'seeing' easier:

Picture of groups	In words	Using symbols
	20 is five groups of four	$20 = 5 \times 4$
	20 is four groups of five	$20 = 4 \times 5$

Students working at this level could then present their findings in pairs to the rest of the class, critiquing each other's work as needed and suggesting what needs to be changed. Teachers should continue to use the words 'groups of' with students until they are sure students are connecting these words with the 'x' symbol. Ask students to tell you what different groups can be made from 8, 14, 9, 22 etc.

Other activities for building arrays could include:

• Have students draw the different types of muffin trays they could use to bake a dozen muffins or 20 muffins; have them write the number of rows and how many in each row in their tray (e.g. three rows of 4 make 12 muffins).

• Ask students to collect pencils for their group; three each – how many will we need?

• Organise students to work in groups to solve problems such as: we planted our petunias in four rows and there were six rows of petunias. How many petunias did we plant?

• Have students play barrier games in pairs where one child has a picture of an array and describes the array (number of rows and number in each row) to their partner who has to draw or say how many there are.

• Ask students to draw bars of chocolate where the total number of small squares is 24. Ask how many different ways can our chocolate block look like? Is a 12 by 2 bar the same as a 2 by 12 bar? How do we know?

• Have students all stand in the classroom or school yard. When you call a number they have to form groups of that number. Anyone remaining stands off to the side. You then call out 'ARRAY' and students have to form rows, one row per group.

• And similarly, the teacher calls out 'array of four groups of six' and students have to make a four by six array; anyone remaining is out and stands to the side.

From repeated subtraction to division

In the same way that multiplication can be modelled as repeated addition, so division can be seen as repeated subtraction. Understanding of division as a concept begins with sharing (which is often modelled as repeated subtraction). We can share out a given number of objects one at a time to a given number of people until there are none left. Students are generally familiar with this method of sharing from their homes and communities and from learning settings such as day care and kindergarten. Although they have engaged in 'sharing' they don't necessarily understand what is happening except that they each end up with the same amount so it is fair. In fact they are *dividing*, or breaking a quantity into equal groups or parts.

How students read and visualise a word problem will affect how they write it symbolically. For example, if the problem is: 'I shared 20 apples equally among five people. How many apples will they each get?' the symbolic representation will be the same as for that which represents: 'I divided 20 apples into groups of five, and gave one group of the apples to each person. How many people were there?' Both these situations can be represented by 20 ÷ 5, even though the situations are different. Students must engage with the words of division – they need to visualise what is happening in order to translate from the context to the symbolic representation.

Unlike with print literacy and other symbolic representations students have encountered in their learning so far, division is not understood *left to right*. Students need to be taught explicitly, in words, that division is understood *right to left*. For example, 12 ÷ 4 is read as: '12 divided by 4'; but is understood as: 'how many groups of four are there in 12?' It takes a number of lessons in order to teach students explicitly how to read and understand number sentences with a '÷' symbol in them. Working in pairs or groups, students can then complete tables such as the following or develop similar tables of their own with gaps in for their peers to complete.

Problem in words	Drawing	Written as:	Read as:
How many threes in 15?		15 ÷ 3	'Fifteen divided by three'
How many groups of 5 pencils are there in 25 pencils?			
If I have 16 CDs and give two people the same number each, how many will they each get?			
There are 24 lollies in the bag. How many will four of us get if we share them equally?			

For students working at this level, teachers should provide a range of these kinds of problems represented in words, have students draw a picture of the grouping, and get them to write the words showing the operation they are performing. Students should then attempt to write the operation using symbols.

Calculating

A standard algorithm is a procedure made up of a number of steps for carrying out a calculation. It is important to focus on oral, mental and concrete models using manipulatives in the early years, rather than focus on standard algorithms. Algorithms become useful if the numbers are beyond the size manageable using mental or simple written strategies. However, students in these years are generally only expected to calculate with single and double-digit numbers. For example, in calculating 35 + 28 they might think:

- Thirty and twenty is fifty, five and eight is thirteen

- Fifty add thirteen is fifty add ten add three, which is sixty three

If they need to support this thinking by writing things down as they go, it is entirely appropriate that they might write:

- 50 + 13

- 50 + 10 + 3

- 63

Students should be explicitly taught how to develop their own informal written strategies. Teachers do this by 'talking aloud their thinking' and jotting down of numbers and words as they go so that students see that 'this is how you do it'. Teachers can also give students some calculations to do and have them work in pairs or groups to get the solutions. Emphasise that students might use many different ways of doing it and that's okay. Have students share their way with the class later, talking aloud and jotting numbers and symbols on the board as they go and explaining their thinking about why it is right. This supports students in understanding that there are many strategies which can be used and enables them to learn from and with others.

Using money

It is important to note that the verbs used (i.e. the actions required of students) change over the levels of the numeracy continuum. At Level 1 (by the end of Foundation [Prep]) students are to *recognise* and *name* coins and notes. At Level 2 (by the end of Year 2) the verbs are *identify* and *use*. This indicates that students are not required to make complex calculations in their money use. This makes sense since they do not learn decimal numbers until much later. Students should come to understand the worth of coins and can do this by *trading*: one ten-cent coin is worth two five-cent coins, one twenty-cent coin is worth four five-cent coins or two five cent coins and one ten-cent coin, and so on.

At Level 2, students need to know the combinations of coins for simple purchases. This does not mean writing the numerical values of each coin or knowing the fractional values of a dollar. Students know by counting that 100 cents make a dollar and can say 'I have three dollars and fifty cents – enough to buy an ice-cream'. Students will see representations of money in decimal forms in catalogues and price lists for the school canteen and in shops in their community but it is preferable to focus on trading and counting in the years K–2 rather than recording money in decimal forms e.g. $3.50.

Trading activities might include students working in pairs or groups to complete tables such as the two below:

Dollars	Fifty-cent pieces	Twenty-cent pieces	Ten-cent pieces
1	2	5	10

Number of twenty-cent pieces	0	5	10	15	20	25	30	35
Number of dollars	0	1	2	3	4	5	6	7

This level of understanding provides the basis for *proportional reasoning* in later years (vii).

Estimating before calculating

We should expect students to estimate the results of calculations first before they use any written method or procedure. Even though mental estimating involves some mental calculation, it is the lack of precision that makes it 'an estimate'. Students need to be taught that they don't need to be completely right when they are estimating; they merely use some mental calculating to give them a sense of the *type* or *size* of answer they are expecting. To estimate, students have to *choose* the operation they are using (if it is not known) and *choose* the numbers they are using (which may or may not involve some rounding to the nearest ten in order to make the actual number easier to work with mentally).

For example, a student attempting the calculation 35 + 27 would be expected to understand both the size of the numbers and the operation indicated by '+' so well that they could be expected to think:

- That's about 35 add 30 so I'm expecting about 65

They then perform the exact calculation using the following thinking:

- Thirty and twenty is fifty, five and seven is twelve

- Fifty add twelve is fifty add ten add two, which is sixty two

If they need to support this thinking by writing things down as they go, they might write:

- 50 + 12

- 50 + 10 + 2

- 62

They are then able to compare their estimate (65) with their exact calculation (62) and know they are likely to be right because this is roughly what they were expecting. Teachers reinforce the idea that an estimate is okay as 'a rough guide' by using words such as: 'That's about right; it's close to'. Students sh:ould be taught to estimate first, then calculate the exact answer (if it is needed) and then compare their estimate with their exact answer.

Monitoring and assessment

All of the above tasks and problems can be used as assessment opportunities as well as for teaching. Understanding of these concepts involves a lot more than doing algorithms or getting a 'right' answer. Assessment tasks and problems should involve an expectation of reasoning and explanation. Students can be asked to draw, write, talk about or show with materials their solutions to problems which involve estimating and calculating with whole numbers.

Monitoring involves keeping a record of whether students can work with different problems types and the numbers they are confident to work with. As students' progress into Years 1 and 2, teachers will also monitor the strategies students use to solve problems and work mentally with numbers.

If students don't appear to be progressing it is important to take a close look at the assessment tasks and their alignment with the intended learning and to consider the explicit teaching or re-teaching which may be required to support individual students. What strategies might be needed what can be modelled by the teacher and or peers and what materials or visual supports (e.g. number lines) might be useful?

Diagnostic tools such as those listed in the Appendix support teachers in understanding where students are in their learning about calculating with whole numbers and provide valuable data to inform teaching.

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers. Teachers can use data from these questions, along with the NAPLAN Toolkit to inform their planning and teaching.

Links across the curriculum

In Kindergarten students engage with mathematical ideas as they explore their environment and interact with adults in intentionally planned play-based learning contexts. For example, a focus on leaves and plants in the outdoor environment might include counting, sorting and recording with an iPad how many leaves of different colours, sizes and textures have been collected. Adults may question the students about how many brown ones they have and what might happen if they found two more. They also encourage students to explain their collections. Such activities link with EYLF Outcome 5: 'Students use language to communicate thinking about quantities, to describe attributes of objects and collections and to explain mathematical ideas' (p. 42).

In Foundation (Prep) to Year 2 other learning areas often provide a context for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? What maths are we using here?*

There are many *Australian Curriculum* numeracy links across the curriculum. In Science lessons when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. Numbers are needed for counting, measuring, comparing, and ordering in the Arts and Physical Education. In History students are learning to apply their knowledge of numbers used to order from first to last (using number lines/timelines). In such instances, classroom teachers would point out to students that 'we are using some maths here'.

Questions for reflection

Do I know the connection between subitising and counting, and subitising and partitioning?

Do I understand the benefits for estimation and mental calculation of being able to partition numbers?

How confident am I with this skill? Can I model it to my students?

Do I model the importance of using thinking as the first choice in calculation over written methods and digital technologies?

How much time do I focus on mental strategies in my class? Is it enough when I consider that this strategy is the one my students will likely need the most in life?

Do I understand the importance of mental calculation for students understanding mathematical concepts?

Do I understand why teaching standard algorithms too early might be harmful for my students?

Do I understand the connections between addition and multiplication, and subtraction and division? What about the mathematical relationship between addition and subtraction, and multiplication and division? Do I know how to teach these connections?

Do I understand why *money* is singled out as a context in the numeracy continuum? What does that mean for emphases in my program?

B. Recognising and using patterns and relationships

The second organising element in the numeracy general capability is 'Recognising and using patterns and relationships'. 'This element involves students identifying trends and describing and using a wide range of rules and relationships to continue and predict patterns. Students apply their understanding of patterns and relationships when solving problems in authentic contexts'.

Some teachers may wonder why *recognising and using patterns and relationships* is important in numeracy or for that matter, mathematics. Mathematics is the science of patterns (viii) and patterns are used to help us organise and make sense of the world in which we live. We therefore need to be able to *generalise* about these patterns rather than having to study each separate pattern. 'Mathematics brings to the study of patterns an efficient and powerful notation for representing generality and variability, and for reducing complexity – algebra' (ix).

There are *patterns* in our number system - 0,1,2,3,4,5,6,7,8,9, for example — that help students learn to count. There are patterns in our lives (Monday, Tuesday, Wednesday...) that help us organise our events and our work. We can learn the words of songs more easily, by considering the patterns in the words. In mathematics, we can learn to count more easily by recognising the patterns that exist in our number system. *Patterns can be related* to each other in ways that can help us study the patterns regardless of the specific elements in the pattern. This is why being able to generalise is so important; it is about asking: 'What is happening here that has nothing to do with the *specific* numbers used?' In a pattern like 4, 8, 12, 16 we can say: *it is growing by 4 each time* and this *generalisation* would be same if the number pattern were 3, 7, 11, 15 or 29, 33, 37, 41.

Key messages

One of the keys to teaching students in K–2 how to *recognise and use patterns and relationships* as the building blocks for more formal learning is to ensure that they:

- have deep understanding about patterns and can generalise about them

- can recognise and describe the variation (or 'change') in relationships.

Patterns can be repeating patterns such as #,#,$,#,#,$,#,#,$... or 3,2,4,1,3,2,4,1,3,2...which are particularly useful in teaching students to generalise. Or they can be growing patterns such as #, ##, ###… or 2, 4, 6… which underpin relationships and functions. If we represent patterns using numbers we can make it easier to both *see* a pattern and to *work with it.*

Links to the curriculum

The *Early Years Learning Framework* (EYLF) gives priority to patterns and relationships in Outcome 5 'Children are effective communicators', and gives evidence examples such as: Children notice and predict the patterns of regular routines and the passing of time; children begin to recognise patterns and relationships and the connections between them; children begin to sort, categorise, order and compare collections and events and attributes of objects and materials, in their social and natural worlds (p. 43).

The content descriptors in the Number and Algebra strand of the *Australian Curriculum: Mathematics* designate what students are expected to be able to do year by year.

The links between 'patterns' and the curriculum frameworks are described in the table.

EYLF	*Australian Curriculum: Mathematics* Content Descriptors		
Kindergarten	**Foundation (Prep)**	**Year 1**	**Year 2**
Outcome 5 *Children are effective communicators:* *Children begin to understand how symbols and pattern systems work* **Outcome 4** *Children are confident and involved learners*	**Extracts from Number and Algebra Strand**		
	• Sort and classify familiar objects and explain the basis for these classifications • Copy, continue and create patterns with objects and drawings	• Investigate and describe number patterns formed by skip counting and patterns with objects	• Describe patterns with numbers and identify missing elements
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
	• Students connect events and the days of the week	• Students continue simple patterns involving numbers and objects	• Students identify the missing element in a number sequence
	***Australian Curriculum* Numeracy Continuum**		
	Level 1a	**Level 1b** Typically by the end of Foundation (Prep)	**Level 2** Typically by the end of Year 2
	Recognising and using patterns and relationships		
	• Recognise simple patterns in everyday contexts	• Describe and continue patterns	• Identify, describe and create everyday patterns

The progression in teaching young children about patterns is that they firstly *recognise* them, then talk about and *describe* them, *continue* existing patterns, *create* their own, and ultimately *generalise* about them.

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

1. Teaching students to have deep understanding about patterns and to generalise about them

Children enter school with understandings about pattern even if they don't yet have the language to describe them. They recognise repeating patterns in their behaviours, events and their environment and might say things like: 'Every morning I get up and get dressed, then I have breakfast'. Note that *recognise* is different from *identify* at Level 2. At Level 1 they are not expected to be able to tell you about the underlying form of the pattern e.g. AAB, AB AB etc. but should be able to describe it by saying red, blue, red blue….

In school, students can build pattern cycles without formalising what they are doing. For example, they might see a pattern such as ball, block, ball, block, ball…and put in the next object of the pattern – a block – and some students could even tell you that the next one is a ball. Deep understanding comes from the idea that a pattern such as AAB, AAB can be made with any materials or be represented in any form.

Teachers need to know that there are two types of patterns (repeating and growing) that are referred to in both the *Australian Curriculum*: *Mathematics* and Numeracy Continuum, although they are not explicitly distinguished.

By the end of Level 1 in Numeracy (Foundation [Prep]) students are expected to be able to describe patterns using words and continue them. They should be able to say things like: 'the pattern goes white, black, red, white, black, red' and when given some more coloured pens can continue the pattern till they run out. By the end of Level 2 (Year 2) they can create patterns of their own given a group of objects or colours. They can also do this with sounds and actions, and if prompted, can show the same repeating

pattern in a different form e.g. *pen, pen, pencil, pencil*; a pattern in sounds can be shown in actions as *stamp, stamp, clap, clap*.

2. Teaching students to recognise and describe the variation (or 'change') in relationships

The emphasis in recognising and using patterns and relationships in K–2 is primarily on patterns as indicated in the Numeracy Continuum Levels 1 and 2. However, if students are to understand trends – as described in Levels 3–6 of the continuum – they need to understand the relationships *between* elements of the pattern and the *number* of the elements in the pattern. They should be taught the mathematical language of patterns: *elements*, *repeating pattern*, *growing pattern*, *sequence*, *cycle*, *predict*, and so on.

Some activities for developing understanding about pattern and relationships

Aim: students recognise patterns in their everyday lives, talk about them and draw them

Conduct a brainstorm with children about the repeating patterns we see and experience around us. Talk about day and night, wet and dry, and so on. Ask children to share what they do when they get up in the morning and the order in which they do it. Encourage them to share ideas about how the year is broken up: birthday, Christmas, Easter, holidays and so on. Students could be asked to draw their repeating pattern as a frieze.

Focus questions might include:

- *How often do we put our clothes on? Every day? Every hour?*

- *Do we come to kinder/school every day? Which days don't we come?*

- *Do we eat lunch every day? What else do we do every day? How often?*

Aim: Students learn to continue repeating patterns

- Give students the first four elements of a number of patterns (on paper or digitally) of pictures of different coloured dogs and ask them to identify the patterns they see. Have them continue the pattern by pasting the correct coloured dogs in the correct order across a line. They can check each other's patterns.

- Ask students to create another pattern with the dogs but starting by using no more than four dogs. Then, their partner has to identify the pattern and continue it, to be checked by the creator of the pattern.

Cover up elements of the pattern and ask questions such as:

- *What goes in here?*

- *Which two things go in here?*

- *What colours will the next four dogs in the pattern be?*

- *Can you describe the pattern?*

Aim: Students learn to identify patterns in sounds and to represent them in words

- Have students work in pairs to make a pattern out of sounds using their own actions. For example they might stamp their feet twice, clap their hands once, and then click their fingers. They then choose one word to describe each action and say their pattern as they make the sounds. For example, saying 'stamp, stamp, clap, click'. Level 2 students might write the words as well and hence represent the pattern in another form.

Focus questions:

- *How many times do we need to hear the pattern before we can tell what it is?*

- *Is it easier to know what the pattern is if we hear it than if we say it?*

- *What other ways could we show this pattern?*

- *How would you describe or explain this pattern? (Encourage them to say things like: 'it is a pattern that repeats and there are four sounds in the pattern')*

Aim: Students learn to recognise and continue growing patterns

Provide students with a pile of small objects such as pattern blocks, paper clips and beads. Have them make a pattern by placing one of the first object at the top of the page, two of the second underneath and three of the third under that. Have them count how many objects in each row, pointing at each row as they say it.

Focus questions:

- *Is the pattern repeating or growing? (Does it go back to 'one' again?)*

- *What is the difference between each row? What is happening from one row to the next?*

- *Point to a row and say: 'How many are in this row?'*

- *How many will be in the fourth row? How do you know?*

Aim: Investigate growing patterns in a hundreds square

- Give students a hundreds square and a coloured pencil or felt pen. Ask them to skip count by fives and colour in each number as they say it. Have them look at the pattern they have created with their colours and describe it by talking about the 'white' squares between the coloured ones. Have them look at the coloured squares and ask them if they can see any patterns in them.

Focus questions:

- *How many white squares are between each coloured square?*

- *How many squares are in the pattern?*

- *What is the last numeral in each coloured square? Is there a pattern in these numerals?*

- *What is the first numeral in each coloured square? Is there a pattern in these numerals?*

Aim: Investigate growing patterns made of shapes, and represent them using numbers in a table

- Give students about 20 square shapes and 15 triangles (pattern blocks are ideal). The teacher makes the first three figures of this pattern (on an electronic white board or on the mat) numbering them:

Students are asked to make the next figure in the pattern, in pairs using their own shapes. They look at the figures made by other pairs and talk about why they are right or not. They can work together to fill in the table:

Figure	1	2	3	4	5
Number of triangles	1	2	3		
Number of squares	0	1	2		

Focus questions (students can make the figures if they need to):

- *How many squares will there be in the figure that has five triangles? How did you work this out?*

- *How many triangles will there be in the figure that has five squares? How did you work this out?*

- *How can you work out how many squares there will be if you know how many triangles there are?*

- *How can you work out how many triangles there will be if you know how many squares there are?*

- *Test your rule to find how many squares there will be if there are eight triangles.*

- *If there are 20 shapes in your figure, how many will be triangles and how many squares?*

Students build patterns and are asked 'What comes here?'

Using pattern blocks or coloured cards/cubes build a pattern with students such as red, red, blue, yellow, red, red, blue... having no fewer than three full cycles of the pattern. Cover up the last few elements of the pattern using paper cups or a page and ask students: 'What comes here?'

Students take turns saying what they think is covered and why they think so. They should be able to say, for example: 'I looked at the colours of the pictures I could see and then looked at what came after that colour last time'.

Monitoring and assessment

Most of the ideas presented here in activities can be used to assess the understanding and monitoring of pattern and relationships. Note that understanding of these concepts needs to include higher-order reasoning about pattern and relationship; assessment must go well beyond continuing and repeating given patterns. You might give students shapes and coloured cubes for example, and ask them to *create* a pattern given parameters. You will need to ask children to justify their reasons for which elements are missing or 'which one comes next', for example. They can do this verbally or using recorded words as they get older. They could explain what is 'wrong' with a section of pattern, telling you why and describing what needs to change to fix it. Monitoring this would involve keeping a record of whether they can recognise, continue, describe or create – in that order – as they progress, focussing support on the actions they are unable to master.

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers. Teachers can use data from these questions, along with the NAPLAN Toolkit to inform their planning and teaching.

Links across the curriculum

In the EYLF, Outcome 5, Key Component 'Children begin to understand how symbols and pattern systems work' (p. 43) there are many examples of how educators promote this learning. For example: 'Educators provide children with access to a wide range of everyday materials that they can use to create patterns and to sort, categorise, order and compare'.

In Kindergarten, teachers will create and foster play-based learning opportunities for students to explore ideas of pattern and relationships and use and extend their mathematical ideas. For example, a basket of musical instruments might be set up in the outdoor area and teachers/other adults could model sound patterns. Students could be asked to explore and make their own sound patterns with the instruments and play them for their peers.

Threading activities which are often used to develop fine motor skills also provide many opportunities to make, continue, talk about and explain patterns. For example, at snack time, students can create and describe the patterns they make by threading fruit onto bamboo skewers.

In the years Foundation (Prep) to Year 2 other curriculum areas provide context for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? What maths are we using here?*

There are many numeracy links across the *Australian Curriculum*. Although it helps to use shapes and manipulatives to teach pattern in the classroom, teachers should also encourage students to see patterns in nature (e.g. shells, flowers, webs), in the built environment (e.g. brickwork, paths, crossings), in books (e.g. structure of a book and in numbers. In Science lessons, when students are learning inquiry skills, they might see the usefulness of generalising patterns (e.g. you plant seeds, they germinate, they grow a bit each day, they develop fruit and so on) so that they don't have to describe what happens for each single plant. In History students are learning to apply their knowledge of time by sequencing objects and events (using number lines/timelines) and identifying patterns in history from the past, present and future (sorting and ordering time and dates). In English students

are learning about words and concepts about print including the patterns used to organise texts such as page numbering, tables of content, headings, chapters, and so on. Teachers can also draw students' attention to patterns that occur in the spelling of words, such as 'madam, level, radar, kayak'.

Questions for reflection

- Do I know what number patterns are and why they are included in the Mathematics Curriculum and Numeracy Continuum? Could I explain this to another teacher?

- Do I understand what students are learning when they can represent one pattern in different forms or representations (e.g. the same pattern in actions, beads, drawings)

- How might I support all of the students in my class to describe a pattern?

- Can students show me one pattern in another form?

- Do students know the difference between a growing pattern and a repeating pattern?

- How might APPs and other ICTs support students in understanding patterns and relationships?

- How can I make links across the curriculum to support students in recognising and using patterns and relationships?

C. Using fractions, decimals, percentages, ratios and rates

Decimals, percentages, ratios and rates are all different forms of fractions. They are also developmentally cumulative, so that students learn about *common* fractions first, then about *decimal* fractions and then about percentages – which are a particular type of fraction, having a denominator of 100. They then learn about ratios and rates.

Key messages

Teachers of K–2 students should realise the *vital importance* of the underlying fraction ideas discussed here for later years of mathematics learning.

To use fractions accurately, we need first to understand them. It is therefore critical that the idea of *equal* quantities or parts is explicitly taught to students during early fraction lessons. This is because most young children come to school with an everyday understanding of fractions that recognises parts or pieces of a whole *but they may not realise that fractions have to be equal parts*. For example, Mum might say 'have half each' to two children or 'have a

quarter each to four brothers and sisters. This does not mean 'measure it exactly and make sure you get the same amount each' it usually just means 'share it as evenly as you can'. It is appropriate in the context of the home or community to use this 'common' terminology. However, since students bring this understanding to the study of fractions at school, teachers need to explicitly build on the knowledge of *halves* being two parts and *quarters* being four parts i.e. the mathematical idea of *equal* parts.

Links to the curriculum

Links to *The Early Years Learning Framework* are to Outcome 5: Children are effective communicators, and in particular where children begin to use symbols to express, represent and communicate ideas (p. 43). In the years before and in the first years of school, children will learn and understand what *wholes* are and that these can be broken into *parts of wholes*.

These understandings are similar for the *Australian Curriculum: Mathematics*. Teachers provide many different types of opportunities for students to learn words about 'whole' and 'equal parts' – try not to use only objects with symmetry (e.g. oranges, pizzas, cakes) but use objects, foods and collections that are irregular when teaching students what a 'whole' and a 'part of a whole' is. You might continue to use words like *half* and *quarter* that students are familiar with from home and it will help them to associate these words with numbers *two* and *four* in the terminology they use at home.

EYLF	*Australian Curriculum: Mathematics* Content Descriptors		
Kindergarten	**Foundation (Prep)**	**Year 1**	**Year 2**
Outcome 4 *Children are confident and involved learners* *Children contribute constructively to mathematical discussions and arguments*	**Extracts from Number and Algebra Strand**		
	• Have 'common' understanding of the words' half' and 'quarter' (author's inclusion)	• Recognise and describe one-half as one of two equal parts of a whole	• Recognise and interpret common uses of halves, quarters and eighths of shapes and collections

EYLF	Australian Curriculum: Mathematics Content Descriptors		
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
		• Students identify representations of one half	• Students divide collections and shapes into halves, quarters and eighths
	***Australian Curriculum* Numeracy Continuum**		
	Level 1a	**Level 1b** • Typically by the end of Foundation (Prep)	**Level 2** • Typically by the end of Year 2
	Using fractions, decimals, ratios and percentages		
	• Recognise a 'whole' and 'parts of a whole' within everyday contexts	• Recognise that a whole object can be divided into equal parts • Identify quantities such as more, less and the same in everyday comparisons	• Visualise and describe halves and quarters • Solve problems using halves and quarters

By the end of Year 1 students need to have learned through explicit teaching the concept of *equality* or *equal amounts*. Parents and teachers might adjust their language by now saying '*you* cut and let *him* choose' and engage in discussions about *fairness* (Outcome 2, EYLF) if one person gets a piece that is bigger than the other.

Other key ideas of fraction in K–2 are:

• Students should learn to think of and describe a half using the words one of two equal parts (of a whole). "When we split something into two equal sized parts we say we have halved it and that each part is half the original thing" (x).

• Students should learn to use a whole, half and quarter of objects and shapes as their benchmarks, being able to recognise if another shape is the same, less or more.

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

In K–2, the focus is on explicitly teaching and building understanding of the words whole, half and quarter rather than the mathematical symbols for fractions; students in K–2 should not be expected to use the symbolic representation of fraction. Students should learn to hear and listen for the words, say them, and use them when describing objects and shapes that are exactly halves and quarters.

Teachers need to teach students explicitly to listen to, hear and make the sounds in the words *half* and *quarter*. To learn what a 'whole' is teachers might use any object, cut or break it into parts, and then put it back together again (if possible) to show the *whole* again. Students should be able to visualise the whole and the parts.

Teaching 'equal' parts is best done through sharing. Make sure you explicitly teach that equal shares is about quantity, not appearance. Be careful not to assume that all students have the same understanding of *fairness*; in some cultures older children and adults are given a larger portion.

Students with a deep understanding of one half will recognise these shaded portions as one half of the whole:

one half
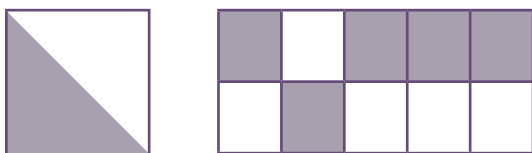
- *They learn this by saying 'one out of two equal parts' or 'one out of every two equal parts'*

one quarter
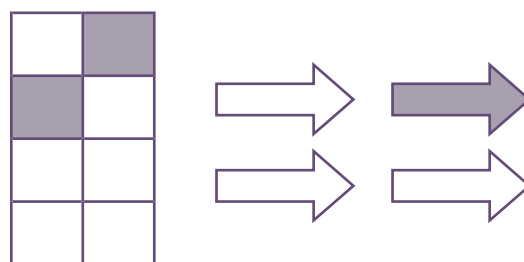
- *They learn this by saying 'one out of four equal parts' or 'one out of every four equal parts'*

Some activities to develop these understandings over the four year period (in increasing levels of difficulty) might include:

- Give students a range of materials (including some items of food) and ask them to hold up above their head items that you call out and say what they are holding up:

  - *A whole apple*   *'I am holding a whole apple'*

  - *A whole piece of paper*   *'I am holding a whole piece of paper'*

  - *A part of a cracker*   *'I am holding a part of biscuit'*

  - *A whole leaf*   *'I am holding a whole leaf'*

- Hold up a whole circle and then hold up parts of a circle (of the same size). Ask students: Is it more than the whole? Is it less than the whole? Why do you think it is more than the whole? Is it the same amount? Who thinks it is more? Who thinks it is less? Why do you think it is less than the whole?

- When students have their fruit at morning play, deliberately cut some of the apples/oranges into less than equal amounts. Have students collect their fruit and then ask them to tell you: Who has half? Who has less than half? Who has more than half? How do you know?

- Bring a large chocolate bar to class and ask the students: How might we cut it in half? Draw it on the board and ask students to come and put a line where they think a half is, and to justify their choice. Give them square paper with 20 chocolate bars drawn on it and

ask them to work in pairs to colour their bar in different ways to show how many ways they can show halves of the bar. Have them mark the page of another pair of students, ticking if correct and drawing a circle around those that are incorrect. Make sure they can tell you why it is wrong!

- Have students count in quarters; start with one whole and count the quarters — one quarter, two quarters, three quarters, four quarters. There are four quarters in a whole. There are two halves in a whole. How many halves/quarters are there in two wholes? How many halves/quarters are there in two and a half? Symbolic representation is not yet used with students at this level.

Give students real life problems and some abstract problems using shapes and objects. The problems should be given in words. Have students work in pairs or groups to solve the problems.

- There are four oranges in a basket. 'How many halves (quarters) of oranges are there?' Draw them.

- Ask: 'If I have three sausages and cut each one in half, will I have enough to give each of my seven friends half a sausage? Why or why not?'

- 'I have four pizzas. I cut each pizza in quarters. How many quarters do I have?'

- 'My Mum gave me three and a half tomatoes for lunch. How many half tomatoes (quarter tomatoes) did I have? If I gave my friend half of the tomatoes that my Mum gave me, how many tomatoes did I keep for myself?'

-

Monitoring and assessment

All the above tasks and problems can be used for assessment as well as teaching. Children can be asked to individually find solutions and to draw and write about why they know they are correct. You might also give them a questions with a hypothetical student's response, and have them mark it: *Are they right? How do you know? How would you help them with this problem?*

There are a number of useful diagnostic tools (see Appendix) which have tasks and questions to support teachers in understanding where students are in their early understanding of fractions.

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers. Teachers can use data from these questions, along with the NAPLAN Toolkit to inform their planning and teaching.

Links across the curriculum

In Kindergarten, teachers will set up play-based learning contexts which enable students to use mathematics purposefully and build on their current numeracy understandings. For example, adults might place particular containers and tools in the sandpit to encourage talk about how many of a particular container will fill another. This provides a deliberate opportunity to discuss the words *whole* and *half* and to make connections to measurement ideas such as *more* and *less, big, small* etc.

In the years F–2 other learning areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here?* Or, *what maths are we using here?*

There are many *Australian Curriculum* numeracy links across the curriculum, including in Science lessons when students are learning inquiry skills they might use contexts that generate numerical data over time e.g. growth of plants. To do this students will need to know how to compare numbers including fractions, and put them in order.

D. Using spatial reasoning

'Using spatial reasoning' is the fourth organising element in the Numeracy learning continuum. 'This element involves students making sense of the space around them. Students visualise, identify and sort shapes and objects, describing their key features in the environment.'

Links to the curriculum

Links to *The Early Years Learning Framework* are to Outcome 3: Children have a strong sense of wellbeing. In particular, 'children demonstrate spatial awareness and orient themselves, moving around and through their environments…' (p. 32). Young children develop ideas about space, shape, balance and angle as they move around the world and build towers of blocks, for example. They will also engage with the other EYLF outcomes through the pedagogies used in assisting them to learn and understand the ideas of what things are and where they are in space and location.

Key messages

In this organising element, students need to learn to:

- visualise 2D shapes and 3D objects

- interpret maps and diagrams

In order to 'see' shapes and objects it helps to think about their features. These *features* include lines, edges, corners, and angles. In teaching and learning about the features it helps if teachers and students have a shared language to be able to talk about them. Therefore using the names of shapes like *square, circle*, and *triangle* and of objects – including some mathematical objects like *pyramid, cone and sphere* – is helpful, as is knowing words such as '*angles*' to describe the features of shapes and objects.

The second idea – interpreting maps and diagrams – is concerned with *where* things are rather than *what* things are. If we can *interpret* a map or diagram we can describe how to get from one place on the map to another. This means that students need to learn and use the language of direction, position and location required to do this.

EYLF	Australian Curriculum: Mathematics Content Descriptors		
Kindergarten	Foundation (Prep)	Year 1	Year 2
Outcome 3 *Children have a strong sense of wellbeing*	**Extracts from Measurement and Geometry Strand**		
	• Sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment • Describe position and movement	• Recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features • Give and follow directions to familiar locations	• Describe and draw two-dimensional shapes, with and without digital technologies. Describe the features of three-dimensional objects • Interpret simple maps of familiar locations and identify the relative positions of key features
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
	• Students use appropriate language to describe location • They group objects based on common characteristics and sort shapes and objects	• Students describe two-dimensional shapes and three-dimensional object • They use the language of direction to move from place to place	• Students recognise the features of three-dimensional objects • They order shapes and objects using informal units. They interpret simple maps of familiar locations • They explain the effects of one-step transformation
	***Australian Curriculum* Numeracy Continuum**		
	Level 1a	**Level 1b** • Typically by the end of Foundation (Prep)	**Level 2** • Typically by the end of Year 2
	Using Spatial Reasoning		
	• Sort and match objects according to their features • Demonstrate awareness of position of self and objects in relation to everyday contexts	• Sort and name simple 2D shapes and 3D objects • Follow directions to demonstrate understanding of common position words and movements	• Identify, sort and describe common 2D shapes and 3D objects • Give and follow directions on maps and diagrams of familiar locations

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

I. *Some activities for visualising 2D shapes and 3D objects from K–2*

Initially students need to be explicitly taught the *language* they need in order to recognise and describe shapes in different positions and orientations. They look closely at everyday objects such as cups or cartons and learn to visualise what they look like from different positions. They think about a stick of celery or a carrot and what it will look like along the cutting line when cut. In order to do these things students need to engage in a broad range of activities that are rich with language and 'what if' questions asked by their teacher.

Initially students will just engage with the shapes and objects; touching, turning, taking apart (if possible, feeling and focussing on specific parts as well as on the whole). Over a number of weeks students learn the specific words for 2D shapes and 3D objects in their environment or drawings including: square, circle, triangle, rectangle, sphere or ball, pyramid, cone.

Students should be given many opportunities to fold and cut paper; cutting corners from folded paper for example, and seeing what shapes result when they unfold the paper and then describing these shapes. They should also use their visualisation skills to 'see in their mind's eye' and describe what a shape or object will look like if it is flipped or turned. Teachers provide activities such as:

- Students call out all the objects they can see in the room. With the teacher they 'sort' them into groups based on whether they have straight edges, flat faces, round faces, sharp points and so on. They can sort based on 'box-shape' or 'bowl-shape', 'flat' or 'bumpy', 'straight' or 'round', and so on. Your goal is to teach as many words that describe spatial features as possible.

- The teacher holds a jug or puts it on the table in the midst of students. Ask them what they can see. Encourage them to look at the handle, spout, base, sides, surface, inside. Model for them how to ask questions and encourage them to be as explicit with their

answers as possible. You might say: Is the handle round or pointy? Is the base or bit that it's sitting on flat or wobbly? – How do you know? Are the sides smooth or rough?

- Bring a scooter into the classroom and ask students to tell you: What are the parts of the scooter? What shapes are the parts? Are there any straight lines? Which lines are straight and which ones are round? Are there any cylinders?

- Play a barrier game where the teacher places an unknown object behind the barrier and students have to ask questions of the teacher about what features the object has. Teachers can only answer yes or no. The winner is the child who guesses what it is. Extend this game by having students play it in pairs.

- Give students in pairs a page with different 2D shapes drawn on it including triangles, circles, squares, rectangles. Have students work together to put a line around those that look the same and to be able to tell you the reason they look the same. Repeat this activity using pictures of 3D objects.

- Give students a page with a list of shapes either named or described (e.g. triangle, shape made of four straight lines) and have them use digital technologies to draw the shapes. They should get into groups or pairs to mark each other's drawing and say why they meet the criteria or not.

2. *Some activities for interpreting maps and diagrams from K–2*

In order to interpret maps and diagrams students need to know the language to describe them. *Awareness of position* for young students is about knowing *where they are* in relation to other things. Students need to be explicitly taught words and terms such as *behind, in front, next to, below, above, underneath, beside, left, right, furthest away, closest, nearest.* When students have learned this language, they use it to describe and locate objects in their classroom and then in the school or immediate environment.

As students become familiar with locating things around them they can then learn to follow directions and move somewhere else based on the words they are hearing. The teacher might also help them to *visualise* directions and think about where they might finish up. Students then need to learn how to give directions to get someone else to move to a familiar place. The teacher will then also need to teach students the four compass points so they can give directions in terms of a north point on an arrow.

- Have students stand as a group in front of you. Say everyone stand next to someone you like. Everyone stand behind a person who is taller than you. Stand in front of a girl if you are a boy. Who is furthest away from Fred? Who is closest to Susan? Put your cap underneath you. Who is beside you, Alan? Who is on your left, Sharon? Do this activity regularly so students learn to use the words quickly and without hesitation.

- Locate objects and things in your classroom, saying things like: Does anyone have something underneath their chair? What is next to the door? What is on the right of the whiteboard? Where is the light; above us or below us? Who is sitting nearest/closest to the door? Who is furthest away from me? Who is sitting beside Ruby?

- Arrange students out in the school yard in a grid pattern, all facing the front where the teacher is standing facing them. The teacher says: John, I want you to move two places forward and then one place to the left. Where will you end up? Toby, if you move one place behind you and three places to the right, where will you be standing? Who is standing there now? Lachlan, how many places forward and then right will you need

to move to finish where Joey is now? If everyone in the back row moved four places forward and two places left, who will be the furthest from me? Who is closest to me now? Which way will he have to move and in what direction to be furthest from me? Who is closest to Sam? Mia, how would you get to where Simon is standing? Julia, how would you get to the position that Janet is standing in?

- Play 'Simon says' with the class, giving directions, e.g. Simon says move two steps back and one right. Simon says move three steps left and then ten steps forward.

- Give older students a simple map of the classroom (like the one below) and have them work in pairs to follow directions that you give them verbally and write down their answers. Directions might include: On the map of the classroom what is in front of the teacher's desk — a chair or the whiteboard? What is next to the window — a desk or the cupboard? What is in front of the whiteboard — the cupboard or the tap?
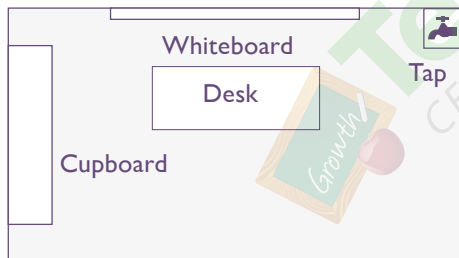
- Give older students a map of a show ground with a north point on it. Ask questions like: What is north of the Ferris wheel? What is east of the big dipper? Is the clown tent east of the entry gate or west of it? Which direction would you need to go in to get from the Ferris wheel to the big dipper? Which direction would you need to go in to get from the clown tent to the big dipper? Ask them to make up some of their own questions using compass points and give them to each other to follow.

Monitoring and assessment

All the above tasks and problems can be used for assessment as well as teaching. If students don't appear to be progressing it is important to it reflect on your assessment tasks; are the questions too hard? Do they assess the learning that you planned for? How might the next task be structured to differentiate the learning for students based on what has been demonstrated?

There are some useful tools for diagnostic assessment of early number listed in the appendix which will provide valuable data to support planning to address particular student needs.

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers.

Teachers might for example, copy questions related to spatial reasoning onto cards and distribute to groups of students asking them to discuss how they might solve the problems and report back to the class. This is an opportunity for teachers to understand the possible misconceptions students hold and plan to address them.

Links across the curriculum

In Kindergarten, teachers intentionally plan and create play-based learning context for students to explore, use and extend their mathematical ideas. For example, to support early ideas of location and arrangement, students may be read the book *Going on a Bear Hunt* and then act it out, constructing their own 'props' in the playground.

Play opportunities with large construction blocks also provide many opportunities for teachers and other adults to model the language of shape and location and challenge students' thinking by asking challenging questions such as: Can you balance the thin block on top of this cube? What happens if you add three more rectangular blocks to this pile?

In Foundation (Prep) to Year 2 other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? What maths are we using here?*

In History students are learning to apply their knowledge of shapes and objects and their ability to sort and classify based on this knowledge. They identify and compare features of objects from the past and present (using words of shape). Geography, the Arts and Technologies also provide multiple opportunities to link with this organising element.

Questions for reflection

Do students in my class know how to visualise something? How do I know?

Have I explicitly taught them how to visualise something? Have I modelled this?

Can they draw what they see 'in their mind's eye'? How do I know?

Can they describe an object using technical language appropriate for their age? What words and phrases do I need to model and focus on? What words and phrases might I need to learn?

Can they compare shapes and objects using technical language appropriate for their age?

Can they recognise and name simple 2D shapes and 3D objects in the environment?

Can they follow directions to get somewhere?

Can they give simple directions to explain to someone how to get somewhere?

Can I make links across the curriculum to using spatial reasoning?

E. Interpreting statistical information

Interpreting statistical information' is the fifth organising element in the Numeracy general capability. It involves students gaining familiarity with the way statistical information is represented…and evaluating data displays of various types.

Key messages

There are two key ideas in the Interpreting statistical information organising element that are important for numeracy:

- interpret data displays

- interpret chance events

In the first idea, interpret data displays, the verb *interpret* is the key; to interpret data displays does not mean you have to be able to collect and organise data or draw the display. Whilst it helps if you know a little bit about collecting, organising and displaying data, these skills are not essential for interpreting it. The second idea – interpreting chance events – is concerned with interpreting information about the *likelihood* of something happening.

Words of chance occur in the homes of most students. They include words and phrases like *maybe, perhaps, might, could;* most students grow up hearing these words and get a sense about likelihood by hearing them used in situations and circumstances. Students need to be taught what phrases like *small chance* or *big chance* mean so that they can make informed decisions when taking risks. The complexity and depth of understanding increases as we have greater experiences with risk in our lives.

Links to the curriculum

Links to the EYLF are to Outcome 5: Children are effective communicators, and in particular children will learn and understand about the ways of presenting information and what things mean when presented in different forms *chance and risk and likelihood* are all about. Children will also engage with the other outcomes of the EYLF through the pedagogies used by their teachers in facilitating the learning of the ideas of data and chance.

Links to the *Australian Curriculum: Mathematics* are evident in the content descriptors and in the relevant organising element of the numeracy learning continuum.

EYLF	Australian Curriculum: Mathematics Content Descriptors		
Kindergarten	**Foundation (Prep)**	**Year 1**	**Year 2**
Outcome 5 • *Children are effective communicators* • *Children begin to understand about the ways of presenting information and what things mean when presented in different forms, and of what chance and risk and likelihood are all about* • *Children begin to understand key literacy and numeracy processes…(Component 2: Children engage with a range of texts and gain meaning from these texts p. 41)* **Outcome 4** • *Children are confident and involved learners*	**Extracts from Measurement and Geometry Strand**		
	• Answer yes/no questions to collect information	• Represent data with objects and drawings where the objects or drawing represents one data value. Describe the displays **Chance** • Identify outcomes of familiar events involving chance and describe them using everyday language such as 'will happen', 'won't happen' or 'might happen'	• Create displays of data using lists, tables and picture graphs and interpret them • Identify practical activities and everyday events that involve chance. Describe outcomes as 'likely' or 'unlikely' and identify some events as 'certain' or 'impossible'
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
	• Students answer simple questions to collect information	• Students describe data displays. They collect data by asking questions and draw simple data displays. They classify outcomes of simple familiar events	• Students make sense of collected information. They collect data from relevant questions to create lists, tables and picture graphs. They describe outcomes for everyday events
	Australian Curriculum Numeracy Continuum		
	Level 1a	**Level 1b** • Typically by the end of Foundation (Prep)	**Level 2** • Typically by the end of Year 2
	Interpreting Statistical Information		
	Interpret data displays • Display information using real objects or photographs and respond to questions about the information displayed	• Recognise how to ask and answer simple data questions and interpret data in drawings or picture graphs **Interpret chance events** • Recognise that some events might or might not happen	• Collect and describe data on a relevant issue based on one variable and display as lists, tables or picture graphs **Interpret chance events** • Identify and describe familiar events that involve chance

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

Interpreting data is the culmination of the collecting, organising and displaying of data – it is the *purpose* for engaging in these activities. Technology can be increasingly programmed to collect, organise and display or represent data, but it is difficult to program it to effectively interpret the data. Note that you cannot learn to interpret data by engaging in the skills of collecting, organising and displaying. For example, people don't learn what graphs mean by drawing them. The skill of reading and interpreting graphs *that someone else has drawn* is the most important skill for the majority of the population.

Whilst the focus is on interpreting data in graphs and tables, some collecting and organising is helpful but the activity must shift to interpretation as the prime goal of the learning. Students need to know that data for graphs comes from real life situations. They should collect information about themselves to display in their first graphs. They might want to know in which months of the year the students in their class have birthdays. The teacher models for them how to find out and record the information and then shows students how to put the information in a display where they can *see* their own data point in a table or in a chart (i.e. one dot in the table or physical display on the floor comes from *them* and represents *their* birthday).

Some activities for interpreting data displays K–2

- Teachers ask data questions such as: Let's find out what sort of lunches children have today? How many types of lunches have we got: sandwiches, roti, rice, fruit. Can we find out how many children bring each type of lunches? Everybody draw a picture of your lunch and we will decide which column to put it in. Draw a table with four (five, or six) columns, one for each type of lunch suggested. Have each child put their picture in one of the columns. Ask children questions about the display: What is the type of lunch that most children have? What is the type of lunch that few/not many children have?

- Have students make similar graphs for information about favourite pets, birds, food, the first letter of children's name, etc.

- Use an electronic white board to make graphs and displays, with students choosing a shape or object to represent them. Collect the data about favourite colours, favourite games, hair/eye colour etc. into columns and show how these columns can be made easier to read by putting the objects on top of each other or in columns or side by side in rows. Ask questions about the data and have students make up questions about the data for the rest of the class to answer.

- Use sticky notes to represent data e.g. the letters students' names begin with, and make a 'graph' on the carpet. Talk about how the data might be organised, asking: Were there any children whose names begin with 'X'? How can we show this? Which letter is used the most as a first letter? How do we know? Are there any letters used the same number of times?

Some activities for interpreting chance events K–2

It is important for teachers to know that chance isn't about probability. In fact, it's the other way around; probability is about chance. Probability is about measuring likelihood. Most people in their everyday lives aren't concerned with measuring the likelihood of something happening or not. They are satisfied to have a general *sense* about likelihood – this is demonstrated in the language we use when talking about chance.

In Foundation (Prep) students need to learn phrases such as *might happen*, and *might not happen*. They learn this difference through events that occur in their lives and they learn the words and phrases to describe how sure we are about events happening by hearing them used in discussions about the events. Even very young children begin to look for explanations so that they can predict things that are beyond their control. They might say: 'Mum *will* pick me up after school today' or 'Auntie Kit *might* pick me up after school today' and this demonstrates that they know that 'will' and 'might' indicate how sure they are. These statements will be based on their life experiences. They might learn by experience that Mum *always* picks them up when it's raining, so there is no doubt about that – they are *certain*.

Teachers support students learning how to interpret chance events when they:

- Model the words of chance by using them in appropriate contexts. You should teach the difference between guessing and being sure. Put a small object like a bottle top under a cushion and say: Who knows where the bottle top is? Is it under the cushion or is it on the floor? Are you sure? Why are you sure? Who knows where the teddy is? Is it under the cushion? How do you know? Are you sure? Did anyone see me put it there? Talk about the fact that you can't be sure or certain unless you saw me put it there. If you didn't see me put it there and you tell me it's under the cushion you are just guessing.

- Play 'Is it possible' games, making statements like: An elephant will walk in the door; Frank will get a piece of paper; Sara will turn into a frog; Mrs (Principal) will come through the door. Have students say whether they are sure/certain it will happen, sure/certain it won't happen, and whether they are not sure but it might.

- When you read stories to students ask them: What might happen next? Who thinks this might happen next? Who thinks it won't happen? Why?

- Put some coloured tiles or plastic caps in a hat. Say to students: I'm going to take one cap out. What colour might it be? Who thinks it will be red? Who thinks it will be blue? Are we sure?

-

Monitoring and assessment

Assessing the interpretation of data K-2 involves giving students lots of practice with different types of displays including tables and charts. Children need to be explicitly taught some strategies for doing this in step by step and holistic ways since a data display can be a very complex text for children – and adults – to read.

Level 2 has children collecting and describing their own data displayed in tables, lists and picture graphs. *Interpreting* isn't included, however children need to be able to *describe* their display. That means they have to be able to tell you what it shows but not be expected to make inferences or say *why* it shows what it does or make comparisons between data categories.

For example, they might ask everyone in the class what their favourite pet is and make decisions with the teacher's help about how to show their data in a tally table such as:

Cat	ЖЖ I
Dog	Ж I
Fish	II
Bird	Ж

They should be able to describe the data in the table saying six children like fish best; eleven children like cats; and answer a question from the teacher like *How many children like dogs?*. They would not be expected to make comparisons such as *more children like cats than dogs or birds are more popular than fish*. This does not mean that you wouldn't ask them questions like *How many children like dogs more than fish?* But they are not expected to be able to do this by the end of Year 2.

In assessing whether they have attained the expected standard for their year of study you would assess their ability to *Collect and describe data on a relevant issue based on one variable, and display their data in lists, tables or picture graphs*.

In assessing whether children recognise that some events might happen and some might not happen you 'might' give them a list (on paper or orally) of events and ask them to say which of them will happen, which will not happen and which might happen. For example:

Write **won't, will,** or **might** in the spaces for each event:

1. It .... rain on Friday

2. I .... go home tonight

3. My Dad........................ turn into a dog

4. My house.................... be made of gold and so on.

You might do the same task, worded differently using words like *impossible*, *unlikely*, *likely* and *certain*.

To assess whether children can identify and describe events that involve chance you might make up sentences and ask them to tell you if they not sure i.e. student says or writes *yes*, *no*, or *maybe*

1. It will snow on your birthday

2. You will have fish and chips for tea

3. The spider on your pillow has eight legs

4. Your cat can fly.

You might then ask them to think of and describe or write an event like the ones above, that have an answer of *yes*, *no*, or *maybe*.

If students don't appear to be progressing it is important to reflect on your assessment tasks. Are the questions too hard? Do they assess the learning that you planned for?

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers.

Links across the curriculum

In Kindergarten, teachers deliberately plan for play-based opportunities for students to explore use and extend their mathematical ideas about statistics and probability. For example, a large pile of natural materials (shells, stones, leaves, gumnuts, pine cones etc.) might be tipped onto a blanket in the Kindergarten room and students asked to sort them out and talk about how they have grouped them. Evidence of students' thinking can be captured as audio using a phone and photos taken, this is the beginning of classification, sorting and the representation of data.

In years Foundation (Prep) to Year 2 other curriculum areas provide contexts for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here? What maths are we using here?*

There are many *Australian Curriculum* numeracy links across the curriculum. In *Science* lessons when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. You would encourage them to collect their data using tally points or informal measurements, and record their observations in tables or spreadsheets. In History students are learning to distinguish between past, present and future (sorting and ordering time and dates), and they are learning to pose questions about the past from a range of sources and exploring these sources (collecting data).

Questions for reflection

Can students in my class identify when a situation requires a will, might, won't response?

Can they decide when something is certain or impossible and explain in their own words what these words mean?

Are students able to put a simple data display together if each child in the class has one piece of data/information?

F. Using measurement

The sixth and final organising element in the Numeracy general capability is 'Using measurement'. 'This element involves students learning about measurement of length, area, volume, capacity, time and mass.

Key messages

There are two key ideas in the using measurement organising element of the general capability that are essential for numeracy:

- estimate and measure with metric units

- operate with clocks, calendars and timetables

Estimate and measure are the two verbs used in the first idea and their order is important; there is no point in estimating *after* you have measured. If the context you are working in demands an accurate measure, you will estimate *first* so that you have some idea of what measurement to expect. That is, your estimation will give you confidence that you have measured correctly, or it will alert you to having made an error and needing to measure again.

Sometimes an estimation is all that is required in the context in which you are working in. If you don't need to be very accurate and you can tolerate some error, an estimation might be sufficient. As with calculation, it is impossible to estimate without deep knowledge of what you are estimating. So, if you don't have a deep understanding of the attributes length, area, volume and capacity *and* the units used to measure them – in this case, standard metric units – you will not be able to estimate their measurements. To estimate measurements you need to *see them in your mind's eye* and use known measures as benchmarks. For example*, I know what a metre 'looks like' so I can look at the*

length of that rope and use my 'visualised metre' to estimate how long the rope is.

To estimate and measure requires some deep understandings about:

1. What needs to be measured (attributes)

2. Units of measurement (so you can estimate and choose an appropriate tool to measure with)

3. Estimation, using your understanding of attributes and units

4. Measuring, using direct (doing the measuring yourself) and indirect (using a combination of measures or measures given to you) methods.

Although time is an attribute, it is singled out in the Numeracy continuum because it is slightly different from the other attributes of length, area, mass and volume/capacity, and angle; it is not possible to see or heft time – *we experience its passing.*

Links to the curriculum

Links to *The Early Years Learning Framework (EYLF)* are to Outcome 5: Children are effective communicators, and in particular Component 1: 'Children demonstrate an increasing understanding of measurement and number using vocabulary to describe size, length, volume, capacity and the names of numbers'. As well, in Component 4: 'Children notice and predict the patterns of regular routines and the passing of time'. In exploring and investigating measurement, children will also engage with the other EYLF outcomes through the pedagogies used by their teachers as they explore the ideas of estimating and measuring, including with time.

Strong links between this element and the *Australian Curriculum* are evident in Mathematics and in the relevant organising element of the Numeracy continuum.

EYLF	*Australian Curriculum: Mathematics* Content Descriptors		
Kindergarten	**Foundation (Prep)**	**Year 1**	**Year 2**
Outcome 5 *Children are effective communicators* *Key Components 1 and 4*	**Extracts from Measurement and Geometry**		
	• Use direct and indirect comparisons to decide which is longer, heavier or holds more and explain reasoning in everyday language • Compare and order the duration of events using the everyday language of time • Connect days of the week to familiar events and actions	• Measure and compare the lengths and capacities of pairs of objects using uniform informal units • Tell time to the half hour • Describe duration using months, weeks, day and hours	• Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units • Tell time to the quarter hour, using the language of 'past' and 'to' • Name and order months and seasons • Use a calendar to clarify the date and determine the number of days in each month
	Extracts from the *Australian Curriculum: Mathematics* Achievement Standards		
	• Students group objects based on common characteristics and sort shapes and objects. They explain the order and duration of events	• Students order objects based on lengths and capacities using informal units. Students explain time durations. They tell the time to the half hour	• Students order shapes and objects using informal units. They tell time to the quarter hour and use a calendar to identify the data and the months including in seasons
	Australian Curriculum **Numeracy Continuum**		
	Level 1a	**Level 1b** • Typically by the end of Foundation (Prep)	**Level 2** • Typically by the end of Year 2
	Using Measurement		
	• Use informal language and/or actions to describe characteristics of length, temperature, mass, volume, capacity and area in familiar environments • Sequence familiar actions and events in a variety of ways	• Measure by comparing objects and indicate if these measurements are the same or different • Sequence familiar actions and events using the everyday language of time	• Estimate, measure and order using direct and indirect comparisons and informal units to collect and record information about shapes and objects • Read digital and analogue clocks to the half and quarter hour • Sequence events by months and seasons and identify a date on a calendar

Planning

Teachers and educators use the tabled information to plan by backward mapping from the curriculum outcomes. According to the level the students are operating at, teachers refer to the EYLF and/or to the *Australian Curriculum*, using the achievements standards and the content descriptors of the Mathematics curriculum and the expected indicators in the *Australian Curriculum* Numeracy Continuum to set instructional goals.

The following activities describe the sorts of learning opportunities teachers need to create in order to develop the required, stated learning in the table, both for numeracy and for the mathematics learning that underpins it.

Putting it into practice

1. Understanding attributes

Part of being numerate in measurement is understanding *what* needs to be measured in a context. It is therefore important that students understand measurement attributes. An attribute is a quality that and object has, for example, its length, height, mass and area. If we want to measure something we are interested in how much of a particular attribute it has.

Once we have decided *what* to measure we need to think about the purpose or *why* we want to measure it, since this helps us decide how accurate we need to be.

2. Understanding units

It is impossible to estimate measures without understanding the units that we use for measurement. Although the element of the Numeracy continuum is '*estimate and measure with metric units*', young students need to estimate and measure with non-standard units before they can do these things with metric units.

Measuring requires us to work out how many *repeats of the unit* are needed to match the thing to be measured. This is essentially what measuring is about. So the understandings needed to measure using units include:

a. Some objects work better than others as units to measure with (due to gaps and overlaps)

b. The bigger the unit chosen to measure with, the smaller the number of repeats

c. For comparison you need to use the same unit of measurement

d. The purpose for measuring tells us which unit will give the most accuracy and how much care is needed in measuring

e. Standard units are no more correct that non-standard units; it depends on the context.

It is helpful if we choose a unit to measure with that relates well to the attribute we are measuring. For example we wouldn't use a pencil

to measure mass, although a pencil would be a good unit if we were measuring length. Students should be engaged in talking about the choices of units to measure with and be able to justify their choice based on the attribute they are measuring.

Understanding the units we use is helped if we can *visualise* the units; that is have a picture of them in our head. When students are able to visualise a centimetre for example, they should have no difficulty estimating the length of their finger or the size of a small computer screen.

3. Estimating measures

The link between understanding attributes and units and estimating, becomes clear when you consider what is needed in order to estimate. Clearly you can't estimate how much of an attribute something has if you don't understand the attribute or the units used to measure it. For example, how can you estimate the area of a carpet if you don't understand what *area* is and if you don't have a mental picture of the *unit* used to measure it, such as a square metre?

Estimation is an 'informed guess'. It is making an approximation based on the information available to you; what you can see, heft, or experience (in the case of time). Estimation is helpful when it is difficult to actually measure something. Confidence in our estimates will improve over time if we practise visualising units.

4. Measuring (direct and indirect)

Direct measuring requires us to work out how many repeats of the unit we use are needed to match the thing to be measured; we do this ourselves rather than using a formula or measures that someone else has taken. Young children need to learn language of comparisons such as taller, tallest, shorter, wider, longest, longest, same as, holds more, and holds less, in order to compare the attributes of their objects. They can and should be taught these words during directed play. Language that needs to be explicitly taught and learned includes:

Attribute to compare	Language needed	Suitable units	Unsuitable units
comparing length	longer, shorter, longest, shortest, same length	piece of string, ruler, pop stick, hose, paper-clip, straw, leaf, pipe-cleaner	ball, apple, box, stone, biscuit
comparing width	wider, widest, narrower, narrowest, same width	piece of string, ruler, pop stick, hose, paper-clip, straw, leaf, pipe-cleaner	ball, apple, box, stone, biscuit
comparing height	tallest, taller, short, shorter, same height	piece of string, ruler, pop stick, hose, paper-clip, straw, leaf, pipe-cleaner	ball, apple, box, stone, biscuit
comparing area	more surface, less surface	piece of paper, book, envelope, tea-towel	cup, bowl, apple, ball, plate
comparing mass	heavier, heaviest, lighter, lightest, same weight	anything that can be hefted/lifted in one hand	anything that can't be hefted/lifted in one hand
comparing capacity	holds more, holds less, holds the same	cup, jug, bucket, spoon	(anything solid or that can't be filled)
comparing volume	takes up more space, takes up less space	package, box	toy truck
comparing angle	pointier, straighter	'square' corner book	(anything round with no angles)
comparing times	more time, less time, faster, slower, quicker	metronome, tocker, egg-timer	-

Indirect measuring requires us to work out how many repeats of the unit we use are needed to match the thing to be measured, and we use it when the object or shape is large or complex. Finding the area of a paddock or park, for example, would be too laborious or time-consuming to use a model of a square metre and lay it out over the paddock repeatedly to find how many square metres in its area. When this is the case we might use a formula made up of a number of different measures (e.g. area = length x width). In learning to measure, young children will not use formulae. They will, however, use indirect measuring by combining direct measures and comparing and ordering attributes. For example, they might compare the lengths of a range of objects they can move by placing them alongside each other. If the objects are too large or fixed in position they can compare by using a third, movable object, as a 'go-between'.

Some activities for estimating and measuring with metric units K–2

- In play-based learning we use comparative language to teach students words like bigger, smaller, wider, heavier, shortest, longest and so on. Students compare lengths, widths, heights and 'heaviness' by hefting and holding one object in each hand. They can learn comparative language about temperature by talking about hotter, warmer, cooler and colder. They can compare capacities by filling containers with water and saying: this one holds more, holds less….. They can compare the duration of time by saying: this will take longer, is quicker, take less time, take more time. Teachers draw attention to these words when students use them as well as modelling and using them. Immerse students in this language, using it frequently and assessing the understanding.

- Have students form pairs and stand back to back. Ask other students: who is the tallest? Who is the shortest? Can we look at the other students and find someone who is shorter than Sally but taller than Sean?

- Bring two or three different sized buckets to class and have students use smaller containers to fill the buckets. Ask: How many cups of water do we need to fill each bucket? Which bucket holds the most? Can we tell which one will hold the most before we fill it? Now let's look at some different sized cups. Can we tell by looking which one will hold the most? Why? Do you think any of the cups will hold the same amount? How could we check?

- Have students use a square or rectangular stamp and see how many times they can use the stamp to cover an A4 page. Ask: How many times did you need to use the stamp to cover the paper? Were there any gaps? Were there any overlaps? How could we have had fewer gaps? How many stamps might we need to cover this big paper (A3)? Do you think it might be twice as many? Why? The same number? How could we find out?

- Form small groups where students work out which book is heavier. They take it in turns to lift one book in each hand and order the books from heaviest to lightest just by hefting. Each child takes a turn to check each other's result. Ask: Are there any books that you think are the same weight? Why? (Teach students explicitly that heaviness is another word for weight).

- Invite everyone to take a cup and get into groups of three. Designate whose cup should hold the most or the least. Use water (or rice) to see whose cup holds the most and whose cup holds the least. Ask students about whether their 'guess' before they measured was right or not and why.

- Have students stand behind a line and throw a beanbag one at a time. Ask: Who threw their bag the furthest length? Who threw their bag the shortest length? Are there any throws that were the same length? What could we use to measure how long each person's throw was? Why?

- Show students how a balance scale works without using it to accurately measure but just to get a sense of which is heaviest and which is lightest by watching which end goes down the most.

- Have piles of uniform (the same) objects on a desk. Give students a table such as the one below on a piece of paper and have them work in pairs/small groups to complete the table (this task can be done to connect with learning descriptors in Section E).

Length of object					
Object to Measure	Number of pop-sticks used	Number of pipe-cleaners used	Number of paper-clips used	Number of blocks used	Number of hand-spans used
Width of window					
Height of student					
Distance around foot					
Length of mat					

Debrief the activity when each group has finished, asking questions such as: *Which was the easiest unit to use to measure the height of the desk/length of mat etc.? Why? Did you have a problem measuring around a foot using pop-sticks/ pipe-cleaners/blocks? What would be a better unit to use? How many pop-sticks did you need to measure the width of the window? Did you use more paper-clips to measure the height of the desk than pop-sticks? How many hand-spans did you need to measure the length of the map? Why did we all get different answers? Are our hand-spans the same? Why?*

Some activities for operating with clocks, calendars and timetables K–2

Young children often find it difficult to learn about time because they can't see it. They can, however, experience the passing of time and can, learn accurately to estimate it by experiencing how it feels and using familiar lengths of time as 'benchmarks' (e.g. the time it takes to watch *Play School* or to drive to Nana's house).

As with other attributes, time is measured by counting units. Students can use benchmarks such as a minute to compare 'how long it takes' using non-standard measures such as tocks on a tocker or the time taken to empty an egg-timer. Teachers support students learning about time when they:

- Talk about the order in which we do things – this will be connected to discussions about pattern (see Section B). Ask students what they do in the morning before school? What do you do after school? What order do we do things in at school? What do you do on Saturdays and Sundays?

- Have students close their eyes and count to ten and put their hand up when they reach it. Teach students that one minute is 60 seconds. Ask them to repeat the activity, shutting their eyes and counting to sixty, raising their hand when they get to it. Repeat this activity every morning.

- Have a large calendar on the classroom wall and circle each child's birthday. Mark off the day on the calendar every morning so they can see the passing of the days and how we can know what day is. Talk about months as the page is turned; teach the names of the months. Use the opportunities every day to ask students questions such as: How many days until the weekend? How many days until next Tuesday? How many weeks until we sing at assembly.

- Have students work together in pairs or groups to put the months and seasons (on cards) in order – forwards and backwards.

- Teach students to read an analogue clock (be aware some will not have these in their homes). Have students watch the clock for extended periods so they can see the large hand turning all the way around the clock while the small hand moves one position. Draw attention to this every hour and half hour. Frequently refer to the clock during the day, asking: What is the time now? Is it half past one? How do you know? Who thinks it is eleven o'clock? How many more minutes until it is half past two?

- Have two clocks on the wall, one analogue and one digital. Refer to them during the day, asking questions like: How long to go until recess? You have five minutes to finish this task – what will the time be then, Frank? When it is three o'clock on the analogue clock what will the time show on the digital clock?

- Give older students a list of times in writing: half past two, quarter to one, quarter past seven, half past five, three o'clock. Have them write the times using digits and also on clock faces – analogue and digital. Next have them put the times in order from earliest to latest.

Monitoring and assessment

In order to *monitor* the learning against what children are expected to learn, teachers need to refer back to the intended learning in the *Australian Curriculum: Mathematics* or the EYLF. They need to continue to keep an eye on what they want children to learn to remind themselves what their goal is.

When monitoring and assessing the numeracy aspects of measuring it is important that students are assessed when using measurement for a purpose rather than pencil and paper tasks which do not require them to actually measure anything or make choices about which measuring tool is best for the task.

If students don't appear to be progressing it is important to reflect on your assessment tasks; are the questions too hard? Do they assess the learning that you planned for? How might the next task be structured to differentiate the learning for students based on what has been demonstrated?

A good source of assessment questions for children in Year 2 is some of the questions in past Year 3 NAPLAN Numeracy test papers.

Teachers might for example use "a problem a day" during a unit of work focusing on measurement. This involves selecting a relevant question from a past NAPLAN test and using it to model problem solving approaches and encourage students to discuss and share alternate strategies and ways of thinking.

Links across the curriculum

In kindergarten teachers plan for play-based learning experiences which enable young students to explore concepts of measurement as they interact with their environment and the people within it. Teachers and adults model and expose students to the language of measurement as they prepare food ('I'm cutting the fruit into thin slices today' and 'look at this orange, it's a sphere') and as they interact with students in water and sand play. Students explore measurement as they build and use construction materials and as they explore their physical world. Adults select picture books and point out measurement concepts as they read to students. They also pose questions which encourage students to measure to answer questions such as: How long? How tall? How big? How heavy?

Other curriculum areas provide context for the application of mathematics. Teachers need to draw the links by asking students: *Will some maths help here?* Or, *what maths are we using here?*

There are many *Australian Curriculum* numeracy links across the curriculum. In Science lessons when students are learning inquiry skills they might use contexts that generate data over time e.g. growth of plants. Teachers encourage students to use informal units in their measuring such as paper clips to measure how much a plant has grown over night. You would point out to them that 'we are using some maths here', or you might use such a lesson to teach the concept of 'all using the same unit' so we can *compare* which plant has grown the most.

In *History* students are learning to apply their knowledge of time by sequencing objects and events (using number lines/timelines); distinguishing between past, present and future (sorting and ordering time and dates); posing questions about the past from a range of sources and exploring these sources (collecting data); and identifying and comparing features of objects from the past and present (using words of shape). In *English* students are learning that using words of time provides *cohesion* in texts they read and in their writing (e.g. *many years ago, later, finally, and then*). These words join parts of the text together.

Questions for reflection

- Have I taught students to visualise measurement units?

- Can they estimate the number of units in a shape or object just by looking at the shape or object?

- Do students in my class understand what attributes are? How do I know?

- Can they compare how much of an attribute a shape or object has, and use the appropriate language to explain why?

- Do my students know what is and what isn't an appropriate unit to measure an attribute, and tell me why?

- Can students tell me some things in their homes that can be measured, or describe some instances when adults measure at home?

- Can I make links across the curriculum to using measurement?

REFERENCES AND FURTHER READING

References and Further Reading

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APPENDIX 1:
USEFUL RESOURCES

*The following are examples of resources
available to support K–2 numeracy development.*

Key national and state documents

Tasmania's Literacy and Numeracy Framework 2015–2017

Supporting Literacy and Numeracy Success www.education.tas.gov.au/documentcentre/Documents/Supporting-Literacy-and-Numeracy-Success.pdf (staff only)

Good Teaching: Differentiated Classroom Practice www.education.tas.gov.au/documentcentre/Documents/Good-Teaching- Differentiated-Classroom-Practice-Learning-for-All.pdf (staff only)

Good Teaching: Quality Assessment Practices www.education.tas.gov.au/documentcentre/Documents/Good-Teaching-Quality- Assessment-Practices-Guiding-Learning.pdf (staff only)

Good Teaching: Curriculum Mapping and Planning www.education.tas.gov.au/documentcentre/Documents/Good-Teaching- Curriculum-Mapping-and-Planning.pdf (staff only)

The Early Years Learning Framework for Australia www.dss.gov.au/sites/default/files/documents/05_2015 belonging_being_and_becoming_the_early_years_learning_framework_for_australia.pdf

Australian Curriculum: Mathematics www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Australian Curriculum Numeracy General Capability www.australiancurriculum.edu.au/generalcapabilities/numeracy/introduction/introduction

NAPLAN Toolkit https://login.education.tas.gov.au/login.aspx?appID=41&redirect=http%3a%2f%2fnaplan.education.tas.gov.au%2fdefault.aspx (staff only)

National School Improvement Tool www.acer.edu.au/files/NSIT.pdf

Whole school approach

Department of Education Improvement Plan - Improving student achievement through a whole-school approach https://www.education.tas.gov.au/documentcentre/Documents/Improving-Student-Achievement-Through-a-Whole-School-Approach.pdf

Numeracy assessment

Count Me In Too (NSW)	www.curriculumsupport.education.nsw.gov.au/countmein/assesment.html	One-on-one assessment of early number. Highly diagnostic for key ideas in number. Useful for K–3. Freely available from the site.
Assessment for Common Misunderstandings (Victorian Department of Education)	www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/pages/misunderstandings.aspx	One on one assessment of key ideas in number (development of other strands under way) Freely available from this site and through Scootle. Highly diagnostic with teacher advice rubrics on where to go next to target teaching.

	Victorian Early Years Numeracy Interview	www.eduweb.vic.gov.au/ edulibrary/public/teachlearn/ student/mathscontinuum/ onlineinterviewbklet.pdf Available online to Victorian Education Dept. Teachers – not DoE (further information)	One-on-one assessment of key ideas in number, space and measurement. Highly diagnostic and indicates growth points for student understanding. Teacher advice on where to go next for teaching focus.
	Queensland Numeracy checkpoints	www.qcaa.qld.edu.au/9321.html	Freely available from this site. Linked to *Australian Curriculum.* Short tasks which teachers can administer. Includes annotated work samples to support teachers in making assessment judgements.
	I Can Do Maths (ACER)	www.shop.acer.edu.au/acer-shop/ group/CAN	Standardised test with Australian norms. Addresses all areas of mathematics. Requires purchase of booklets etc. Useful for pre and post testing and collection of base line data. Does not include teacher advice on where to target teaching. Commercial product.
Video	**Pattern and Structure Assessment (PASA)** (ACER)	www.acer.edu.au/pasa	Focuses on understanding how students think and reason about important aspects of mathematics. One on one interview with 15 key tasks. Online diagnostic reports to inform teaching can be generated after testing. Commercial product.
	Junior Assessment of Mathematics (JAM) New Zealand	www.nzmaths.co.nz/junior-assessment-mathematics	Freely available. Useful from Year 1. Focuses on understanding how students think and reason about each strand of mathematics. One-on-one interview.
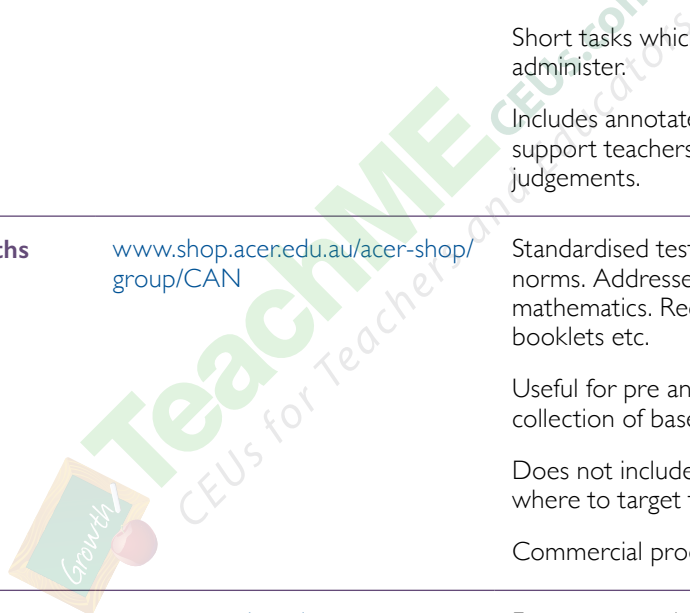
Teaching practices

AITSL: information and videos on quality numeracy practices.	www.aitsl.edu.au/australian-professional-standards-for-teachers/illustrations-of-practice/detail?id=IOP00070
Early Childhood Literacy and Numeracy: Building Good Practice, Marilyn Fleer and Bridie Raban, Australian Government DET, 2014	A booklet for use by professional practitioners in preschools and childcare centres to review their current practice and increase their effectiveness in stimulating young children's developing literacy and numeracy skills. www.docs.education.gov.au/documents/early-childhood-literacy-and-numeracy-building-good-practice-resources-booklet
Educators' Guide to the Early Years Learning Framework, DEEWR 2010	www.docs.education.gov.au/system/files/doc/other/educators_guide_to_the_early_years_learning_framework_for_australia.pdf
TCH: The Teaching Channel	An excellent collection of videos demonstrating effective classroom strategies for numeracy teaching. It has been developed to support the US national curriculum, but has many videos relevant to Australian schools. www.teachingchannel.org/

Numeracy resources

Scootle	Scootle gives teachers access to many thousands of digital curriculum resources they can use to inform their own planning and support their teaching. The resources include learning objects, images, videos, audio, assessment resources, teacher resources and collections organised around common topics or themes. The resources are aligned to the endorsed areas of the *Australian Curriculum*. Logging in: Teachers in Tasmanian government schools log in using their DoE username and password.
TES Education Australia	Access to hundreds of numeracy resources for ages 3-7 relating to *The Early Years Learning Framework* and the *Australian Curriculum: Mathematics* www.tesaustralia.com/
Australian Curriculum **Lessons**	*Australian Curriculum* Lessons is an excellent user-submitted site that depends on teachers to post their great lessons so that other teachers can get ideas and lessons to use in the classroom. www.australiancurriculumlessons.com.au/category/english-lessons/
NRICH	www.nrich.maths.org/frontpage Problem solving tasks and ideas from the University of Cambridge.
Teachertube	A collections of videos, audios, photos, blogs and documents for teachers, parents and students. www.teachertube.com
Top Drawer Teachers	www.topdrawer.aamt.edu.au/ Top Drawer Teachers includes a wealth of ideas on key mathematical ideas including advice on assessment, planning and student tasks from the Australian Association of Mathematics Teachers.
NSW Numeracy Continuum	www.numeracycontinuum.com/ NSW Numeracy Continuum – support in teaching key mathematical ideas.
WA strategies for incorporating numeracy into primary schools	Ideas for including focus on the numeracy general capability in the primary years of schooling. www.k10outline.scsa.wa.edu.au/media/documents/outline_downloads/Numeracy-and-Literacy-in-Primary-School-Teaching.pdf